KTH Mathematics

Exam in SF2863 Systems Engineering
Tuesday June 12, 2012, 14.00–19.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.

No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, in particular if you use a method not taught in the course. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points, including bonus points from the homeworks, is sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.
The order of the exercises does not correspond to their difficulty level.

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1. Frasse has started a new company specializing in cultivating strawberries and apples, where the customers come to pick their products by themselves.

Customers arrive to the system according to a Poisson process with rate 100 customers per hour. Each such new customer will either go to the strawberry field, with probablity p, or to the apple gardens, with probablity $1-p$. The value of this parameter $p \in [0;1]$ can be determined by Frasse by playing suggestive music at the entrance, which means that he can (in some sense) steer the arriving customers.

The strawberry field will be modelled as an $M|M|\infty$ queue, with an exponentially distributed service time with mean 40 minutes, followed by a payment office modelled as an $M|M|1$-queue, with service intensity 100 customers per hour.

The apple garden will be modelled as an $M|M|\infty$ queue, with an exponentially distributed service time with mean 20 minutes, followed by a payment office modelled as an $M|M|1$-queue, with service intensity 120 customers per hour.

A customer who has finished picking strawberries will either go to the apple garden, with probability 0.5 (regardless of how many times he has already been there), or immediatly leave the whole system, with probability 0.5. A customer who has been to the apple garden will either go to the strawberry fields, with probability 0.2 (regardless of how many times he has already been there), or immediatly leave the whole system, with probability 0.8.

(a) For which $p \in [0;1]$ can the system be in steady state? .... (3p)

(b) Assume from now on that $p = 0.2$ is such that the system is in steady state. What is the probability that there are no customers at either of the two payment offices ? .... (3p)

(c) On average, at steady state, how many customers are picking strawberries and how many are picking apples? .... (2p)

(d) Determine the expected number of customers in the system (in steady state). .... (2p)

(e) Determine the expected total time in the system for a customer (which enter the system in steady state). .... (4p)

2. Frasse needs 100 kg (per week) of a certain fertilizer during the forthcoming 4 weeks. He may order the product only on friday evenings. The ordered quantity (if any) is then delivered early monday morning, in time for the planned fertilization of that week. The unit price for the fertilizer is 1000 Euro per kilo. In addition, there is a fixed ordering cost of 700 Euro each time an order is placed (for administration, transportation, et.c.) If Frasse stores some fertilizer from one week to another, the holding cost is 3 Euro per kilo and week. On friday evening just before the first week of the considered planning period the inventory is empty, and it should be empty also by the end of the fourth week.

(a) Calculate an optimal order plan for Frasse. .... (7p)

(b) Assume that the holding cost is changed from 3 to $3 + c$ Euro per kilo and week.What is the largest and smallest (most negative) value of c for which the optimal plan from (a) above is still optimal? What are the new optimal order plans if c just passes these critical values? .... (3p)

3. Frasse is concerned about the safety of his dear strawberries. Every year there is the risk that the crop fails due to lack of nutrients, lack of water, lack of sun and to frost bites. To avoid this Frasse can take measures to decrease these risks; he can add extra fertilizer, extra water, use satellite solar reflectors, and put extra blankets on the fragile plants.

The cost of applying n extra measures of type i is $C_i(n) = nc_i$ USD. Assume that the probability that there is no problem of category i when n extra measures of that that type have been applied is $P_i(n)$. Assume also that for $n = 0, 1, \cdots, 5$ the probabilities are $P_i(n) = p_i e^{k_i n}$.

(a) Formulate a function $f(n_1, \cdots, n_2)$ that describes the probability for success, i.e. a crop with no problem of any category, and formulate an optimization problem for maximizing that probability and with the constraint that the total extra cost should be less or equal to the budget S USD. .... (2p)

(b) Formulate a recursive equation that can be used to solve the optimization problem using dynamic programming. Determine also a function to initiate the resursion with.

Assume $N = 2$, $k_1 = 1/10$, $k_2 = 1/20$, $p_1 = 1$, $p_2 = 2$, $c_1 = 3$, $c_2 = 2$ and $S = 5$. Determine the optimal solution using the dynamic programming recursion. .... (6p)

(c) Can you reformulate the problem so that you can use the theorem on marginal allocation that we prove in the course?

Determine all efficient distributions of extra measures that give a total extra cost less or equal to 10 monetary units. . . . . (6p)

Solving the problem by total enumeration will not give any points.

4. Frasse is thinking about the option of cultivating ecological strawberries. He can obtain a higher price for the ecological strawberries, but the risk of having problems with the crop increases (in the simple model that we use here).

By studying historical data he has observed that if he has a good crop one year, the probability of having a good crop next year is 0.8 if he uses pesticides and chemical fertilizers and 0.6 if he uses ecological manure, otherwise the crop will be bad. If he has a bad crop one year, the probability of having a good crop next year is 0.6 if he uses pesticides and chemical fertilizers and 0.4 if he uses ecological manure, otherwise the crop will be bad. The cost for one years need of pesticides and chemical fertilizers is 600 USD and for manure it is 100 USD. A good ecological crop generates an income of 2000 USD, a good non-ecological crop yields 1200 USD, and bad crops yields 800 and 600 USD respectively.

(a) Formulate this as a Markov decision problem for maximizing the expected revenue (=income -cost) per year.

Use the policy iteration algorithm to *improve on* the starting policy to always use pesticides and chemical fertilizers, if possible. How much will the new policy increase the expected revenue? . . . . (8p)

(b) Now extend the previous model to take into account the effect of building up a good brand reputation of beeing an ecological farmer. Frasse has observed, by studying ecological farmers in another country for a rather short time period, that for every continuous year they use ecological manure they get more goodwill and can add 100 USD to their profit, i.e. for the first year they get 2000 and 800 for a good and bad crop respectively, and the next year they get 2100 and 900 and so on. If they use pesticides one year, the next year they use ecological manure the income will be back at 2000 / 800 USD.

Construct a new Markov model, complete with costs and transition probabilities that only depends on the current state. Define it well.

Would it be possible to apply the Markov decision method on this problem? Motivate well. . . . . (4p)

Good luck!