KTH Mathematics

Exam in SF2863 Systems Engineering
Wednesday December 12, 2012, 14.00–19.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.
No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, in particular if you use a method not taught in the course. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points, including bonus points from the homeworks, is sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.
The order of the exercises does not correspond to their difficulty level.

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1. Frasse and Heathcliff have now run their tech support center for mobile telephone apps called "Eye-phone experts" for one year. The bussiness is doing well, more people are calling, Frasse has increased the success rate and speed of his service and Heathcliff has increased his service rate by asking all customers "what do you think is the problem?" (he realized that they are more likely to solve their problems themselves than he is).

The current state of affairs is defined as follows. Customers are calling the support center reception with intensity 40 callers per hour. At the reception the bureaucratic twins Jo and John are working and it can be modelled as a $M|M|2$ queue with service intensity 24 callers per hour for each server. When the customer have been served at the reception they are with prob. 3/4 transfered to Frasse, with prob. 1/4 transfered to Heathcliff.

The service of Frasse is modelled as a $M|M|1$ queue with service intensity 40 callers per hour and when served by Frasse the customers gets the solution to their problem with prob 4/5 and leaves the system, or, with prob. 1/5 Frasse transfer them directly to (the caller queue to) Heathcliff.

The service of Heathcliff is modelled as a $M|M|1$ queue with service intensity 20 callers per hour and when served by Heathcliff the customers gets the solution to their problem with prob 1/2 and leaves the system, or, with prob. 1/2 Heathcliff transfer them directly to (the caller queue to) Frasse.

(a) Will the queues in the network converge to some stationary state?
What is the average total number of customers in the system? . . . . (5p)

(b) The company charges 4 SEK per minute that a customer is in the system (including queueing time). What is the average cost a random customer calling Eye-phone experts have to pay? .... (5p)

2. Frasse is going to buy christmas gifts to his customers, teddybears with the writing "Happy Xmas 2012". The cost for buying each bear is C dollars. A customer who gets a gift will express his gratitude by generating an extra income for Frasse next year of k dollars. On the other hand, a customer who gets no gift will generate m less dollars of income the next year. The problem is that Frasse can only order the gifts once and at that time he does not know how many customers he should send the gifts to, but he has estimated that the probability distribution for the number of gifts is uniform between zero and N. The bears that are left over he will give to an orphanage generating a goodwill of g dollars per bear.

(a) Define a general expression for the expected profit that Frasse would get if he purchases x bears.

Calculate an optimal order plan for Frasse for the case that $C = 30$, $k = 40$, $m = 20$, $N = 100$ and $g = 10$.

Assume that the probability density function for the demand of the gifts is continuous and constant on the interval $[0, N]$. .... (7p)

(b) Assume that the probability distribution for the demand of the gifts is discrete and constant on the set $\{0, 1, \cdots, N\}$.

Check if the optimal number of bears you obtained in (a), or the closest integer, is optimal for the discrete problem. .... (4p)

For full credits we want a stronger argument than first order optimality condition (derivative equal to zero) to motivate optimality.

3. Frasse wants to develop a new app that determines the optimal choice of apps on the clients smartphone start screen. For simplicity we assume that all start screens are one unit tall and N units wide, where N is an integer. The client has to determine M candidate apps that each will have a width of $w_i \in \{1, 2, 3\}$ and have to assign a utility number u_i that should reflect the clients satisfaction of having app number i on the startscreen. We assume also that if we add an app multiple times it does not increase the utility.

(a) Use the marginal allocation algorithm (and show that the conditions for using it are satisfied) to determine all efficient solutions for maximizing utility and minimizing required space for $N = 0, \cdots, 8$ when $M = 5$, $w_1 = 1$, $w_2 = 1$, $w_3 = 2$, $w_4 = 2$ and $w_5 = 3$ and for a client with utility numbers $u_1 = 1$, $u_2 = 2$, $u_3 = 5$, $u_4 = 6$ and $u_5 = 8$. .... (4p)

(b) Formulate a general optimization problem that maximizes the clients utility under the constraint of a startscreen of width N units. .... (1p)

(c) Use dynamic programming to formulate a recursion for the function describing the optimal value of the optimization problem in (b) for different values of N. Determine also a function to start the recursion with.

Use this dynamic programming approach to solve the problem for $N = 8$ and the data provided in (a). .... (6p)

For full credits you should solve it for $N = 8$, and to avoid too much calculations it is recommended that you translate it to a shortest path problem and solve it graphically. Do not forget to write down the optimal solution.

(d) Would it complicate things to solve the problem, as in (a) and (c) respectively, if the width of the apps were given by $w_1 = 1$, $w_2 = 1.1$, $w_3 = 2.111$, $w_4 = e$ and $w_5 = \pi$? Explain how? .... (2p)

Solving the problem by total enumeration (or other methods than those specified above) will not give any credits.

4. Frasse has developed a new app, a game called "Duopole". It is a big seller and now Frasse wants to increase his newfound wealth by playing the game himself and play optimally.

Duopole is a two person zero-sum board game that look like this:

0	1
2	3

In the game there are two different six-faced dice. We assume that each face of the dice have equal probability to come up.

Die 1, D_1, has three faces with a 1, two faces with a 2 and the last face has a 3.

Die 2, D_2, has two faces with a 1, two faces with a 2 and the last two faces a 3.

The players start by putting their pawns on square 0.

Player 1 then chooses a die, D_1 or D_2, and moves his pawn the corresponding number of steps as the die show. If the pawn passes square 3 it ends up on square 0, *i.e.*, if the die roll is a 2 and the pawn is on square 2 or 3 it will end up on square 0. Player 2 then has to pay player 1 as many dollars as the number on the square on which player 1 ends up on.

Then it is player 2's turn to play. He can also choose between D_1 or D_2, moves according to the same rule, and collects money from player 1 according to where he ends up.

The players then takes turns moving and continues from where they ended up last round. The game is very addictive and we assume that the players are playing a very large number of rounds.

It is clear that optimal strategies of both the players are independent of each other and similar. Frasse can not influence the actions of his opponent so he can only focus on playing optimally himself.

(a) Describe the game-play by a Markov decision process where you only model the movement of player 1 (Frasse). Determine the one-step transition probability matricies corresponding to the different choices of die. Determine also the expected incomes at each pawn position and die choice. .... (4p)

(b) Consider this as a Markov decision problem for maximizing the expected winnings per round (disregarding the losses which can not be controlled).

Formulate the problem to find the optimal policy as a linear programming problem. Define all the variables (what do they mean) and parameter matrices carefully.

Write out the objective function and constraints without summation operators.

How is the optimal policy determined from the solution of the LP? ...... (4p)

(c) Start with the policy using D_2 if the pawn is on square 0 and D_1 otherwise.

Use the policy iteration algorithm to *improve on* the starting policy, if possible. If you were using the initial policy and could choose which square to start from, which one would you choose? .... (8p)

Good luck!