KTH Mathematics

Exam in SF2862 and SF2863 Systems Engineering
Monday June 3, 2013, 14.00–19.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.

No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, in particular if you use a method not taught in the course. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points, including bonus points from the homeworks, is sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.
The order of the exercises does not correspond to their difficulty level.

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1. Frasse and Heathcliff have started their own patent office, the so called F&H patent office. Each patent that comes to their office has to be read by both of them before a decision is made. As the patents arrive they are placed in a pile and on a first-arrived first-served basis the patents then ends up in Heathcliffs hands. It takes Heathcliff an exponentially distributed time to read through a patent, with average time 10 hours. When he has finished reading through the patent it is with probability 0.6 passed on to Frasse and with probabilty 0.2 he realizes that he has misunderstood the whole idea of the patent and has to start reading through the patent again. With probability 0.2 he looses the patent and it disappears forever. (We assume that the second, or third..., reading time will be independent of, and have the same distribution, as the first reading)

When the patents then ends up in Frasses hands. It takes Frasse an exponentially distributed time to read through a patent, with average time 8 hours. When he has finished reading through the patent it is with probability 0.8 classified with pass or fail (based on his and Heathcliffs gradings), with probabilty 0.1 he realizes that he has misunderstood the whole idea of the patent and has to start reading through the patent again, and with probabilty 0.1 he realizes that they both has misunderstood the whole idea of the patent and it is sent back to Heathcliff to start reading through the patent again.

Clients arrive with new patents to their office according to a Poisson distribution with an intensity of 1/20 patents per hour.

(a) Will the queues in the network converge to some stationary state?
What is the average total number of patents in the system? ....(5p)

(b) What is the probability that an arbitrary patent will get lost in the process? (2p)

(c) What is the average time for a patent to pass through the F&H office? . . (3p)

(d) When a patent is sent back for another readthrough above, we assume that it is placed last in the queue again. If we instead assume that a patent sent back is given highest priority, so Heathcliff (or Frasse) will immediately stop reading the patent he is currently looking at, and start reading that patent, and will start with the interrupted patent, as a new patent, once he finish the prioritized paper. How will this change the average queue length? . . . . (2p)

2. The running costs for the F&H Patent office is 100 dollars per day. To finance the business Frasse goes to the local loan shark to borrow money. Each meeting with the loan shark costs 150 SEK, and every time he visits he will borrow the same amount, D dollars. Each day that Frasse has t ($t \geq 0$) dollars left of the money he has borrowed he has to send Heathcliff to the loan shark with $t/100$ SEK.

As you have noticed there are two different currencies used in the company and dollars are used for the finance account and SEK are used for the expenses account.

(a) The finance of F&H has to be secured every day, so the finance acount can not be negative. Assume that Frasse wants to minimize the average total expenses per day.

How often should Frasse visit the loan shark?
(number of days does not need to be an integer)
How much should he borrow each time? (Determine D) . . . . (5p)

(b) Now assume that Frasse decides to include the possibility to use financing by taking a particular kind of SMS-loans. For this SMS-loan the interest rate is 1/10 SEK per day and per dollar borrowed. There is no start-up cost or closing cost of that loan. However, we assume that he can not take these loans very often, so that in practice he will able to take it only once per "cycle".

How much should Frasse then borrow from the loan shark, and how much should he borrow with the SMS-loan? At what moment should he take the SMS-loan?

Determine an expression for the average total cost per day, determine the equation system that defines the solution, and as explicitly as possible determine the optimal solution. . . . . (5p)

3. Frasse is planning for the future of the F&H Patent office. Next year he is imagining that the Patent office is a multiple facility service system with 4 different facilities. They work in parallel and each of them can be modelled as an $M|M|s_i$ model. His plan is to have 8 employes working in the different facilities and he would like to distribute them to the facilities so that the total average waiting time is minimized, *i.e.*, he wants to minimize $W_q(s_1) + W_q(s_2) + W_q(s_3) + W_q(s_4)$.

For the $M|M|s_i$ model the average waiting time for facility i with service rate μ_i and $\rho_i = \lambda_i/\mu_i$ is

$$W_q(s) = \frac{\rho^s}{s!(1-\rho/s)^2\mu s}\left[\sum_{n=0}^{s-1}\frac{\rho^n}{n!} + \frac{\rho^s}{s!(1-\rho/s)}\right].$$

The values needed for solving this problem are tabulated below:

$W_q(s)$	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.7$	$\rho = 0.8$
$s = 1$	4.3103	16.0000	38.5675	68.9655
$s = 2$	0.1988	1.2403	2.4974	3.3501
$s = 3$	0.0083	0.1197	0.3122	0.4556
$s = 4$	0.0003	0.0103	0.0363	0.0594

You do not have to use more decimals than needed in your calculations.

Here, let the arrival intensities be given by $\lambda_1 = 0.01$, $\lambda_2 = 0.025$, $\lambda_3 = 0.035$, $\lambda_4 = 0.04$, and let the service intensity be constant, *i.e.*, $\mu_i = 0.05$ for $i = 1, \cdots, 4$.

(a) Dyer and Proll has shown that the total average waiting time for $M|M|s_i$ systems is an integer-convex decreasing function.

Check if it is true for our function for the tabulated values of s.

Use the marginal allocation algorithm (and show that the conditions for using it are satisfied) to determine all efficient solutions for minimizing the total average waiting time and for a total number of persons ranging from 4 to 8. .....(6p)

(b) In (a) we consider the facilities as independent queues and add the average waiting times. So a long waiting time is not considered worse if there are many constracts waiting instead of just a few.

If we assume that customers come to the future F&H with intensity $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$, and then randomly are divided up to the different facilities so that facility i has arrivals of intensity λ_i; what would be the expression for the mean waiting time then?

If we use this mean waiting time to perform the allocation of employees, what would change?

You do not have to redo all calculations if you did them in (a). It is enough to explain explicitly how they are performed. ....(4p)

Solving the problem by total enumeration (or other methods than those specified above) will not give any credits.

4. As a side business F&H are selling cakes in their office. The cakes are displayed on two pillars designed to hold one cake each. Each day there is a probability of 0.2 that one of their customers buys one cake for 100\$, and 0.8 that they can control themselves not to buy. F&H may buy one cake for 50\$, or two cakes for 80\$, from their local supplier. The cakes have to be ordered at the beginning of a day, before they know if they sell a cake that day, and it is delivered the next morning. If they place an order so that there are extra cakes, *i.e.*, more than two, the next morning, Heathcliff will eat the extra cakes. If there are no cakes and a customer wants to buy a cake then there are no penalty fees, since the customers were not expecting cakes in the Patent office in the first place.

(a) Assume that day 1 there are no cakes, and that F&H wants to maximize the expected profit (= income from sales - cost for purchases) until end of day 3. Solve this using dynamic programming and determine the optimal cake order quantity for each day. (which will depend upon how many cakes have been ordered and sold) .... (6p)

(b) Now solve the infinite horizon case, *i.e.*, determine an optimal policy for the cake order quantity that maximizes the expected profit per day.

Decide on your own starting policy, but do only one step of the policy iteration and then comment about the optimality of that policy. .... (8p)

(c) We have assumed that the cakes are not deteriorating over time. It would be more realistic to assume that the selling value of each cake decreases by, for example, 10% per day.

What modifications in (b) would have to be done to consider this case? Would this correspond to a discounted value function, why or why not? Explain. (But you do not have to solve the modified problem) .... (4p)

Good luck!