KTH Mathematics

Exam in SF2863 Systems Engineering
Monday January 12, 2015, 14.00–19.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.
No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, and always define the variables and notation you use. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points, including bonus points from the homeworks, is sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.
The order of the exercises does not correspond to their difficulty level.

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1. The two up and coming stars of science Goran and Hilda are spending the beginning of the new year correcting exams. We assume that the time for each of them to correct an exam is exponentially distributed with a mean value of 20 minutes. Once an exam is corrected it is passed to the harsh examiner Pedro who at a glance sends 20% of the exams back for recorrection to the same person who corrected it the first time. We model that selection process as completely random, immediate and independent of how many times it been sent back before.

The exams that pass the selection process ends up at Pedros desk which form a queue and the results of the exams are in order of arrival registered in his computer. The time to register the exam is modelled as an exponentially distributed stochastic variable with mean value 5 minutes.

After being registered the exams are sent for scanning and thereby leaves the system. (the scanning process is not part of the considered system)

Assume that exams arrive to Goran and Hilda, from outside the system, according to some independent Poisson processes with arrival rate λ and then by random, each exam is either sent to Goran with 50% probability or to Hilda with 50% probability. Determine λ so that at steady state there is a total of 160 exams handled by Goran and Hilda together.

What is the average total number of exams in the system at steady state?

Determine then the expected time it takes for an exam to pass through the system at steady state.

At steady state how much of the time are each of Goran, Hilda and Pedro idling, *i.e.*, doing nothing at all? And how much of the time are they all idling at the same time? .... (10p)

2. Goran and Hilda are working as teachers for a course where 100 students are coming to the classes. Hilda is an experienced and very consistent teacher, and Goran is working hard to attract students.

There are now three classes to go in the course and Goran has 40 students in his class. You should now help him to find the optimal strategy for maximizing the total number of students he will have in the last three classes (minus the price for any special classes).

If Goran gives an ordinary class 10% of the students will next class switch to Hildas class. At each class Goran can also choose to use a special trick to attract students for his next class. If he offers the students cake at a class he will, instead of losing 10%, get 10 more students for the next class (the ten students come from Hildas class). If he offers a hint about some exam question he will keep all his students and gain 50% of Hildas students for next class. However, there is a price for everything, and buying a cake has the price 150 SEK which he decides to be equivalent to the utility of "15 students", and the price for giving out a hint is equivalent to the utility of "40 students" (no matter if the hint is correct or not there would be consequences if it is discovered).

Define carefully the variables you introduce, and the objective function considered.

Determine a dynamic programming recursion for the optimal remaining objective function.

Solve the problem, determine the optimal strategy. .... (10p)

You may disregard problems with fractional students and any of the classrooms getting more than 100 students or less than 0.

3. Frasses nephew Clyde is studying at the university and Frasse is doing his best to help him to be an efficient student. This christmas Clyde has 3 exams to study for, and he has a maximum of 5 days to study for the exams.

Let s_j be the number of days that he choose to study for exam j. From his vast experience of exams (at least of being the involuntary main character in many of them) he has found out that the probability that Clyde will pass exam j if he spends s_j days studying for it is

$$1 - \frac{(s_j - A_j)^2}{B_j}$$

where $A_1 = 10$, $A_2 = 8$, $A_3 = 12$, and $B_1 = 100$, $B_2 = 100$, $B_3 = 150$.

Use the Marginal allocation algorithm (and show that the conditions for using it are satisfied) to determine all efficient solutions for maximizing the expected number of passed exams when the number of days spent studying ranges in $\{0, \cdots, 5\}$.

When you present the efficient solutions we want both the expected number of passed exams and the distributions of how many days should be spent studying for each of the exams. .... (6p)

4. Frasse also has a long term plan for Clydes whole student life. Being a pragmatic and understanding uncle he wants to maximize the long term happiness of Clyde which he is describing using a utility function. Frasse models the status of Clydes student life as a Markov chain with a binary state where 1 signifies that Clyde passed his last exam, and 0 that he failed it. Clyde takes every exam, and each time he decides whether to study or party before the exam. If he passed his last exam, he will pass his next exam with probability 3/4 if he chooses to study and with probability 1/2 if he chooses to party. If he did not pass his last exam, he will pass his next exam with probability 1/4 if he chooses to study and with probability 1/8 if he chooses to party.

The utility is 10 if Clyde studies, passed the last exam and passes the next one.
The utility is 2 if Clyde studies, passed the last exam and fails the next one.
The utility is 10 if Clyde studies, failed the last exam and passes the next one.
The utility is 0 if Clyde studies, failed the last exam and fails the next one.

The utility is 38 if Clyde parties, passed the last exam and passes the next one.
The utility is 6 if Clyde parties, passed the last exam and fails the next one.
The utility is 20 if Clyde parties, failed the last exam and passes the next one.
The utility is 4 if Clyde parties, failed the last exam and fails the next one.

Start with the policy to always study. Use the policy iteration algorithm (without discounting) to *improve on* the starting policy, if possible.
Is the starting policy optimal? If not, determine a policy that gives a higher expected utility at steady state. What is the expected utility for each exam at steady state for the best policy you have obtained? . . . . (10p)

5. (a) Frasse is running a small bussiness at the university selling noodles to Clydes fellow students. He is supplying 500 students with three packages of noodles a day. Each package costs 2 SEK when he orders them from a web shop. The deliveries are punctual and always delivers in two days and each time he orders there is an ordering cost of 1000 SEK. He has a deal with a local storage where he can keep his inventory for a rent that is 0.03 SEK per package and day.

How much noodles should Frasse order each time, and how often should he order, if he wants to minimize the long term cost per day.

At what level of the inventory should the orders be placed?

You may assume that the quantities handled are large enough so that the demand can be approximated with a fixed constant rate of continuous demand. . . . (6p)

(b) Now assume that Frasse wants to allow back-orders, *i.e.*, he will let the inventory drop to zero and those students who arrive to buy noodles he will give an IOU (I Owe You) ticket instead so that as soon as he gets a new delivery they will get the missing noodles. As an additional compensation he will give them an aspirin-pill for each pack of noodles he owes and each day that he owes it. The cost for Frasse per aspirin is 0.10 SEK.

Determine an equation system with two variables and two equations that can be used to determine the optimal ordering levels. Solve the equation system.

Motivate why and if the profit per time unit is higher in (b) than in (a). (8p)

Good luck!