KTH Mathematics

Exam in SF2863 Systems Engineering
Saturday December 19, 2009, 14.00–19.00

Examiner: Krister Svanberg, tel. 790 71 37

Allowed tools: Pen/pencil, ruler and eraser. A formula-sheet is handed out.
No calculator is allowed!

Solution methods: Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, in particular if you use a method not taught in the course. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points, including bonus points from the homeworks, are sufficent for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.

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1. Consider an aircraft base with a repair shop and an inventory of spare enginees. Aircrafts with a defect engine arrive at the base according to a Poisson process with rate 0.3 defect engines per 24-hour period. When such an aircraft arrive, the defect engine is immediately removed from the aircraft and brought into the repair shop. If the inventory of spare engines is non-empty, a functioning engine is immediately installed into the aircraft which is then operable again. But if the inventory of engines is empty, the aircraft is grounded while waiting for a functioning engine. When a defect engine has been repaired, it is immediately brought to the inventory of spare engines.

The repair shop is modeled as an M/M/1 queueing system, which means that there is a singel repair team, and the repair time is a random variable with an exponential distribution. The expected repair time for an engine, not including possible queueing time, is 48 hours.

Let $s =$ the number of spare engines allocated to the base, i.e. the number of engines in the inventory when there is no engine in the repair shop.

(a) What is the average number of engines in the repair shop? . . . . (2p)

(b) What is the probability that there is at least two enginees in the repair shop (at a randomly chosen point in time). . . . . (2p)

(c) What is the probability that there is at least one grounded aircraft standing at the base (at a randomly chosen point in time)?
Answers should be given for each of the three cases $s = 0, 1, 2$. . . . . (3p)

(d) What is the average number of grounded aircrafts standing at the base?
Answers should be given for each of the three cases $s = 0, 1, 2$. . . . . (3p)

2. A queueing system consist of two service facilities, called A and B. Customers arrive to the system according to a poisson process with rate 6 customers per hour. Each new customer first go to facility A. Customers who have been served at A will either go to B, with probability 0.8, or immediatly leave the whole system, with probability 0.2. Customers who have been served at B will either go to A, with probability 0.5, or immediatly leave the whole system, with probability 0.5. In each of the facilities there is today a single server with exponentially distributed service time with mean 5 minutes for the server in A, and 6 minutes for the server in B.

The management intends to increase the service capacity by employing one more server to work in parallel in one of the facilities. The new server will also have a exponentially distributed service time with the same mean as the already existent server in the facility (5 minutes in A or 6 minutes in B). The question is in which of the two facilities, A or B, the new server should be placed.

(a) Assume that the objective is to minimize the average number of customers in the whole system. What is then the optimal place for the new server, in A or in B? . . . . (4p)

(b) The management also consider the possibility of introducing a fourth server. Their guess is that an optimal allocation then would be two servers in each facility (and not three servers in one facility and one server in the other). Show that they are right, i.e. that two servers in each facility is optimal. (4p)

(c) Assume that the objective instead is to minimize the expected time an arriving customer will spend in the system before leaving. Repeat (a) and (b) above with this new objective. . . . . (4p)

3. A consulting company ABC should do four different jobs for an important customer. ABC has the capacity to allocate in total 11 consultants to these four jobs. The jobs should be carried out on four different places, remote from each other, so no consultant can be assigned to more than one job. Further, at least one consultant must be assigned to each job, which leaves 7 consultants to be allocated. If s_j of these 7 consultants are assigned to job j (in addition to the compulsory consultant already assigned) then the time t_j it takes to carry out job j can be approximated by $t_j = \dfrac{c_j}{s_j + 1}$, where c_j is the time it takes for one single consultant to do the job. The measure used by this particular customer to evaluate the work done by ABC is $T = t_1 + t_2 + t_3 + t_4$. The smaller T, the more satisfied customer.

Data: $c_1 = 18,\ c_2 = 30,\ c_3 = 48,\ c_4 = 66$ days.

Help ABC to allocate the 7 additional consultants in such a way that T becomes as small as possible. Motivate carefully how you can guarantee that your solution is optimal! . . . . (8p)

4. The number of units in a certain inventory is a non-negative integer. The demand is modelled by a poisson process with rate 5 units per day. When the inventory gets empty, it is immediately filled with N units. This replenishment is so fast that the probability that there should be a shortage during the filling can be neglected.

(a) For a given N, the inventory can be modelled as a Markov process, where the states are defined by the number of units in the inventory.
Calculate the stationary probabilities π_j for the different states in this process. Then express the average level of the inventory as a function of N. ...... (4p)

(b) Assume that there is a fixed cost 1000 SEK each time the inventory is replenished, and an inventory holding cost of 1 SEK per unit and day. Formulate a reasonable objective function and calculate an optimal value of N. ...... (4p)

5. By the end of each week (Friday evening), a certain system is in one of two states, *Good* or *Bad*. During the weekend, one of two actions must be chosen, *Standard* overhaul or *Extended* overhaul. For both types of overhaul, the system will be Good on Monday morning but may turn Bad during the week.
If a Standard overhaul is made then the probability that the system will turn Bad during the week is 0.4, while if an Extended overhaul is made then this probability is only 0.2.

The cost for an overhaul depends on the current state. If the system is Good, a Standard overhaul costs 1.000 SEK and an Extended overhaul costs 3.000 SEK. If the system is Bad, a Standard overhaul costs 10.000 SEK and an Extended overhaul costs 14.000 SEK. Moreover, if the system turns Bad during a week, the average cost due to loss in production that week is 5.000 SEK.

(a) Assume that it is Friday evening and that the system should be run only for two more weeks, whereafter we will not care about the state of the system. Determine an optimal decision for this weekend. Do this for each of the two states the system may be in. .... (4p)

(b) Assume now that the system should be run for a long time, and that the objective is to minimize the long-run average cost per week (without any discounting). Use the Policy Improvement method to determine an optimal policy. Start with the policy of always making Standard overhaul. .... (4p)

(c) Since there are just four possible long-run policies in this small example, the optimal policy can be found by calculating, for each separate policy, first the corresponding stationary distribution for the states and then the corresponding average cost per week. Do that! .... (4p)

Good luck!