KTH Mathematics

Suggested solutions for the exam in SF2863 Systems Engineering.
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1. (a) The jobs should be sorted according to WSTP, i.e. with decreasing w_j/P_j quotient.

Here $2/30 = 1/15$, $5/25 = 1/5$, $3/12 = 1/4$, $9/27 = 1/3$, and $4/16 = 1/4$.

So the schudule should be $4, 3, 5, 2, 1$ or $4, 5, 3, 2, 1$.

The sum $\sum_{k=1}^{5} w_{j_k} C_{j_k} = 1200$ for both schedules above

For proof see the textbook.

(b) If all weights are equal to one, then the jobs should be sorted according to shortest job first (SJF). Here we have two machines to use, assume jobs $j_1, \cdots, j_{N_1}$ are scheduled on machine one, and $k_1, \cdots, k_{N_1}$ are scheduled on machine two. Minimizing

$$\sum_{\ell=1}^{N_1} w_{j_\ell} C_{j_\ell} + \sum_{\ell=1}^{N_2} w_{k_\ell} C_{k_\ell}$$

it is clear that all jobs on machine one should be scheduled according to SJF, and all jobs on machine two should be scheduled according to SJF.

The only remaining question is how to determine which machine each job should be assigned to.

Assuming all but the longest job are assigned, it should go last in one of the schedules, then it is obvious that is should be scheduled on the machine that is first free. (by comparing the increase of the objective function value from choosing either machine) This argument can be repeated to schedule the jobs of increasing length on the first available machine.

Here, job 3 is shortest, it can be scheduled on either machine, assume we schedule it on machine 1.

Job 5 is the second shortest, so we schedule it on machine 2.

Job 2 is the third shortest, it is scheduled on machine 1. Since it is available at time 12 and machine 2 is available at time 16.

Job 4 is the fourth shortest, it is scheduled on machine 2. Since it is available at time 16 and machine 1 is available at time 41.

Job 1 is the fifth shortest, it is scheduled on machine 1. Since it is available at time 41 and machine 2 is available at time 43.

Then 3,2,1 are scheduled on machine 1 and 5,4 on machine 2, in that order.

2. (a) Assume that the grain growing in a small area around the collection point is collected and transported to the silo which is positioned at the position $(\hat{x}, \hat{y})$.

Let the transportation costs be proportional to the weight of the transported goods and the distance transported. Assume the distances transported are as the birds fly. Then the transportation costs to the silo are proportional to

$$\int_0^{10}\int_0^{20}\sqrt{(x-\hat{x})^2+(y-\hat{y})^2}C\,dxdy.$$

Considering that the grain is later transported to the mill we add the cost for the transporting the grain there which is proportional to

$$\sqrt{(0-\hat{x})^2+(100-\hat{y})^2}C\cdot 10\cdot 20.$$

The transportation costs between the silo and the mill are probably more efficient, so another cost factor should probably be used here.

The optimal position $(\hat{x},\hat{y})$ is obtained by minimizing the sum of the two costs above. Assuming there is not a lake at that position, roads are available and connected to the mill et.c.

(b) Assume (x_i, y_i) are the locations of the mills and (z_j, w_j) are the locations of the distilleries. Let the variable δ_i be one if mill i is contracted and 0 otherwise. Let c_{ij} be the cost per unit transported from mill i to distillery j, and this is here assumed to be $c\sqrt{(x_i-z_j)^2+(y_i-w_j)^2}+d_i$, where c is a per distance transportation cost and d_i is the cost per unit to the mill. Let K_i be the cost for utilizing mill i. Let u_{ij} be the weight of the product transported from mill i to distillery j.

$$\left[\begin{array}{ll}\min & \sum_{i=1}^5\delta_iK_i+\sum_{i=1}^5\sum_{j=1}^3 c_{ij}u_{ij}\\ \text{s.t.} & \sum_{i=1}^5 u_{ij}=D_j,\quad j=1,2,3\\ & \sum_{j=1}^3 u_{ij}\le C_i\delta_i,\quad i=1,\cdots,5,\\ & \delta_i\in\{0,1\}, u_{ij}\le 0\end{array}\right]$$

(c) In the 25 pubs there are small variations in the consumption of the product, but it can be assumed that they are rather corollated, so that considering the sum of all demand there is still a small variation.

We can expect that the variations in the distribution chain increase for each level we go up, according to the well known bull whip effect. So the distilleries will experience slightly higher variations, and the mills even higher than that.

This might have consequences depending on how close to the capacity of the used mills the production plan is working and more mills may have to be contracted.

3. For alternative 1 the $\lambda T=10$, and the number of engines can only be placed in one location and we should increase the number of engines until EBO is less than 5.

Consider the following table, obtained from the last column of tabulated values of R

s	EBO	cost
0	10.0	0
1.0	9.0	500.0
2.0	8.0005	1000.0
3.0	7.0033	1500.0
4.0	6.0137	2000.0
5.0	5.0429	2500.0
6.0	4.11	3000.0
7.0	3.2401	3500.0
8.0	2.4604	4000.0
9.0	1.7932	4500.0
10.0	1.2511	5000.0

Here we see that 6 engines are required The cost for (10 years of) alternative 1 is then

$$\underbrace{50.000 * 10 * 0.2}_{\text{for repairs}} + \underbrace{500.000 * 6}_{\text{for engines}} = 3.100.000$$

For alternative 2 the $\lambda_1 T = 0.2$, $\lambda_2 T = 0.4$, and $\lambda_3 T = 0.2$. The marginal allocation table is

$R_1(s_1)/c_1$	$R_2(s_2)/c_2$	$R_3(s_3)/c_3$
0.0086466	0.0049084	0.0028822
0.0059399	0.0045421	0.00198
0.0032332	0.0038095	0.0010777
0.0014288	0.0028326	0.00047626
0.00052653	0.0018558	0.00017551
0.00016564	0.0010743	0.000055212
0.000045338	0.00055337	0.000015113
0.000010967	0.00025567	$3.6557 \cdot 10^{-6}$
$2.3745 \cdot 10^{-6}$	0.00010682	$7.9149 \cdot 10^{-7}$
$4.6498 \cdot 10^{-7}$	0.000040661	$1.5499 \cdot 10^{-7}$
$8.3082 \cdot 10^{-8}$	0.000014199	$2.7694 \cdot 10^{-8}$

The number of spare parts should be increased until EBO is less than 5.

s_1	s_2	s_3	EBO	cost
0	0	0	8.0	0
1.0	0	0	7.1353	100.0
2.0	0	0	6.5413	200.0
2.0	1.0	0	5.5597	400.0
2.0	2.0	0	4.6512	600.0
2.0	3.0	0	3.8893	800.0
3.0	3.0	0	3.566	900.0
3.0	3.0	1.0	2.7013	1200.0
3.0	4.0	1.0	2.1348	1400.0
3.0	4.0	2.0	1.5408	1700.0
3.0	5.0	2.0	1.1697	1900.0

Here we see that $s_1 = 2$, $s_2 = 2$ and $s_3 = 0$ is required. The cost for (10 years of) alternative 2 is then

$$\underbrace{20.000 * 10 * (0.05 + 0.10 + 0.05)}_{\text{for repairs}} + \underbrace{300.000 * (10 + 1)}_{\text{for facilities}} + \underbrace{600.000}_{\text{for engines}} = 3.940.000$$

Clearly alternative 1 is the most economical on a 10 year basis.

The cost for an extra engine is 500.000. It would decrease EBO from 4.11 to 3.24, i.e. by 0.87, and therefore in 10 years reduce the penalty cost by $10 * 0.87 * 400.000$ which is much larger than the cost for an engine. So it is economical motivated to buy an extra engine for alternative 1.

For alternative 2 we would invest in one more part 2 for 200.000. It would decrease EBO from 4.65 to 3.88, i.e. by 0.72, and therefore in 10 years reduce the penalty cost by $10 * 0.72 * 400.000$ which is much larger than the cost for the extra part. So it is economical motivated to buy an extra part for alternative 2.

4. (a) First case. This can be modelled as two economic order quantity model, EOQ model.

The demand is $d_1 = 2$, $d_2 = 10$ per day. The holding cost is $h = 3$ SEK per cubic meter and day. The ordering cost is $K_1 = 300$ and $K_2 = 450$.

Then the quantity to be ordered is

$$\hat{Q}_1 = \sqrt{\frac{2d_1K_1}{h}} = 20m^3 \text{ toilet paper}, \quad \hat{Q}_2 = \sqrt{\frac{2d_2K_2}{h}} = 10\sqrt{30} = 54.8m^3 \text{ diapers.}$$

and the cost per time unit is

$$\frac{K_1d_1}{\hat{Q}_1} + \frac{K_2d_2}{\hat{Q}_2} + \frac{h\hat{Q}_1}{2} + \frac{h\hat{Q}_2}{2} + C_1d_1 + C_2d_2 \approx 224.$$

Second case, then $T = \frac{Q_1}{d_1} = \frac{Q_2}{d_2}$ so that deliveries are done simultanteously. The total cost for a period is then

$$c_1Q_1 + K_{12} + \frac{h}{2}TQ_1 + \frac{h}{2}TQ_2 + c_2Q_2$$

where $K_{12} = 550$. Cost per time unit is then

$$c_1Q_1/T + K_{12}/T + \frac{h}{2}Q_1 + \frac{h}{2}Q_2 + c_2Q_2/T = c_1d_1 + K_{12}/T + \frac{h}{2}Td_1 + \frac{h}{2}Td_2 + c_2d_2$$

which is minimized for

$$T = \sqrt{\frac{2K_{12}}{h(d_1 + d_2)}} \approx 5.5$$

and the cost per time unit is

$$c_1d_1 + K_{12}/T + \frac{h}{2}Td_1 + \frac{h}{2}Td_2 + c_2d_2 \approx 199$$

Third case, let $T_1 = nT_2$ where $T_2 = Q_2/d_2$ and $T_1 = Q_1/d_1$. The total cost for a period T_1 is then

$$c_1Q_1 + K_{12} + \frac{h}{2}T_1Q_1 + \frac{h}{2}nT_2Q_2 + nc_2Q_2 + (n-1)K_2.$$

Cost per time unit is then

$$c_1Q_1/T_1 + K_{12}/T_1 + \frac{h}{2}Q_1 + \frac{h}{2}Q_2 + nc_2Q_2/T_1 + (n-1)K_2/T_1$$

$$c_1d_1 + K_{12}/(nT_2) + \frac{h}{2}nT_2d_1 + \frac{h}{2}T_2d_2 + c_2d_2 + (n-1)K_2/(nT_2)$$

which for fixed n is minimized for

$$T_2(n) = \sqrt{\frac{2(K_{12} + (n-1)K_2)}{nh(nd_1 + d_2)}}.$$

We can determine n by minimizing the cost per time unit with $T_2(n)$ inserted. The cost per time unit with $n = 2$ is

$$c_1d_1 + \frac{K_{12}}{nT_2} + \frac{h}{2}nT_2d_1 + \frac{h}{2}T_2d_2 + c_2d_2 + \frac{(n-1)K_2}{nT_2} \approx 205$$

The smallest cost is obtained for alternative 2.

(b) Assume $P_i(k,t)$ is the probability that there is demand of k units of product i within the next t time units.

You have to order new products when the inventory level is such that the probability that there will be shortage is 5%. Let s_i be the smallest number such that $\sum_{k=s_i+1}^{\infty} P_i(k,1)$, the probability of demand larger than s_i, is less than 5%. The demand of each product per day can be estimated as the mean $\sum_{k=1}^{\infty} kP_i(k,1)$ and be used to determine order quanitities.

(c) The three types of inventories are cyclic inventory, pipeline inventory, and safety inventory. Safety inventory is the physical inventory in the shop that is kept to protect against unexpected demand that can cause shortage, Cyclic inventory is the physical inventory in the shop kept to satisfy the mean demand, and Pipeline inventory is the products that have been ordered but yet not received.