KTH Mathematics

Exam in SF2866 Applied Systems Engineering, SF2868 Systems engineering, Business and Management, Part 1
Monday October 26, 2015, 14.00–19.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.
No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, and always define the variables and notation you use. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points are sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.
The order of the exercises does not correspond to their difficulty level.

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1. Given jobs $J_1, \cdots, J_5$ with processing time P_j and weights w_j according to the following table:

Job	P_j	w_j
1	30	2
2	25	5
3	12	3
4	27	9
5	16	4

The jobs will be processed one-by-one on a single machine.

Consider the problem to find a schedule that minimize the weighted sum of completion times, *i.e.*

$$\sum_{k=1}^{5} w_k C_k.$$

(a) First describe the rule that determine which order the jobs should be scheduled in, then apply it on this example, and finally prove that this rule has to be satisfied for an optimal schedule. ....(8p)

(b) Assume that there are now two parallell machines and that all the jobs have weights 1. How should the jobs now be scheduled on the two machines? .(4p)

2. (a) Assume that we have suppliers of grain who generates C tons per year and square kilometer in an area bounded by the points $(0, 0)$, $(0, 10)$, $(20, 0)$, and $(20, 10)$. Now a silo is going to be built and the optimal location of the silo should be determined considering that the grain is later transported to a mill located at $(0, 100)$.

Make necessary assumptions to solve the problem and determine an optimization formulation. .... (5p)

(b) Assume now that there are mills located at $(0, 100)$, $(0, 200)$, $(100, 100)$, $(-50, 100)$ and $(200, 100)$, that are producing a maximum of $C_1, \cdots .C_5$ tons per year. Assume that we have three distilleries located at $(-50, 0)$, $(50, 100)$, and $(-150, 150)$ with demands D_1, D_2 and D_3. Determine an optimization problem that determines which mills should be contracted by the distilleries in order to minimize the cost. The transportation costs can be assumed to be proportional to the distance and weight transported and if a mill is contracted there is a fixed cost for opening up the service and a cost per each ton delivered. .... (5p)

(c) In the nearby town there are 25 pubs and restaurants that consume the products delivered by the distilleries. It can be expected that there is a rather small variation in the consumption. What could be expected from the variations of the flows in the stages considered in part a) and b), why? and what could be the consequences? .... (3p)

3. Consider the procurement of a Helicopter engine maintenance from a life cycle cost point of view. Assume that the time horizon is ten years. To simplify things we consider only one base.

There are two competing offers that should be evaluated.

Alternative 1, there is a possiblity to keep an engine inventory at the base, and repair of engines is done by sending the engine to an external repair shop who will return a working engine in 50 days. There is an engine failure with intensity $\lambda = 0.2$ [engines per year].

Alternative 2, there is a possibility to keep engine parts as inventories at the base. Each engine consists of 3 parts, which fail with intensities $\lambda_1 = 0.05$, $\lambda_2 = 0.1$ and $\lambda_3 = 0.05$ [units per year]. The defect parts are repaired by sending it to the external repair shop who will return a working unit in 40 days. The extraction of the engine parts and the mounting of the parts is assumed to be instantaneous, but require an extra cost of 300.000 SEK per year for renting facilities and the competence, and an extra 300.000 SEK for setting everything up the first year.

Compare the (minimal) life cycle costs of the two alternatives if they are both required to have a total expected number of backorders less or equal to 5. (We only consider efficient solutions to the multiobjective optimization problem minimizing both EBO and costs for spares.)

Assume that cost per engine repair for alternative 1 is 50.000 SEK, and 20.000 SEK the cost per unit repair for alternative 2.

Assume that the investment cost for an engine is 500.000 SEK, and for the units are 100.000, 200.000, 300.000 SEK.

λT	1	2	3	4	5	6	7	8	9	10
k=0	0.63212	0.86466	0.95021	0.98168	0.99326	0.99752	0.99909	0.99966	0.99988	0.99995
k=1	0.26424	0.59399	0.80085	0.90842	0.95957	0.98265	0.9927	0.99698	0.99877	0.9995
k=2	0.080301	0.32332	0.57681	0.7619	0.87535	0.93803	0.97036	0.98625	0.99377	0.99723
k=3	0.018988	0.14288	0.35277	0.56653	0.73497	0.8488	0.91823	0.95762	0.97877	0.98966
k=4	0.0036598	0.052653	0.18474	0.37116	0.55951	0.71494	0.82701	0.90037	0.94504	0.97075
k=5	0.00059418	0.016564	0.083918	0.21487	0.38404	0.55432	0.69929	0.80876	0.88431	0.93291
k=6	0.000083241	0.0045338	0.033509	0.11067	0.23782	0.3937	0.55029	0.68663	0.79322	0.86986
k=7	0.000010249	0.0010967	0.011905	0.051134	0.13337	0.25602	0.40129	0.54704	0.6761	0.77978
k=8	$1.1252 \cdot 10^{-6}$	0.00023745	0.003803	0.021363	0.068094	0.15276	0.27091	0.40745	0.54435	0.66718
k=9	$1.1143 \cdot 10^{-7}$	0.000046498	0.0011025	0.0081322	0.031828	0.083924	0.1695	0.28338	0.41259	0.54207
k=10	$1.0048 \cdot 10^{-8}$	$8.3082 \cdot 10^{-6}$	0.00029234	0.0028398	0.013695	0.042621	0.098521	0.18411	0.29401	0.41696

Table 1: Tabulated values of $R(k)$ for different values of λT.

If there is a fee for backorders that is 400.000 SEK per unit and year, can it be economically motivated to increase the inventory level for the two alternatives?

The recursive formula for determining EBO is given by

$$EBO(s+1) = EBO(s) - R(s), \quad EBO(0) = \lambda T,$$

where $R(s)$ is the probability of shortage given an initial inventory level of s.
. . . . (10p)

4. Consider a shop that sells two products, toilet paper and diapers. Each product has a daily demand of 2 and 10 cubic meters respectively. The storage cost for a cubic meter is 3 SEK/Day. Delivery costs have a fixed set up cost of 200 SEK, and an additional cost of 100 SEK for ordering toilet paper and 250 SEK for ordering diapers.

(a) First, determine the optimal order quantities if shortage is not allowed and the two products are ordered seperately without coordination. (Minimizing total cost per time unit for each product individually)

Second, determine the optimal order quantities if shortage is not allowed and the two products are ordered in coordination, so that both the products are ordered at the same time.

Third, determine the optimal order quantities if shortage is not allowed and the two products are ordered in coordination, so that the second product is ordered at a fraction of the times that the first product is ordered, or the other way around. Describe how the optimal fraction can be determined.

Compare the cost of the alternative strategies. (For case three you may "choose the optimal fraction", different from one.) . . . . (9p)

(b) Assume now that the demand is not deterministic, instead let the demand of the two products be stochastic such that $P_i(k, t)$ describe the probability that there is a demand of k units of product i within the next t time units. Assume that the inventory problem is under continuous review and that the delivery time is one day. How would you approach the problem if you would like to guarrantee that the risk of shortage for both the products is less than 5%? (4p)

(c) In inventory problems we talk about three types of inventories, which are they, and describe how they are defined in this problem. . . . . (2p)

Good luck!