KTH Mathematics

Exam in SF2866 Applied Systems Engineering, SF2868 Systems engineering, Business and Management, Part 1 Friday January 8, 2016, 8.00–13.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.

No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, and always define the variables and notation you use. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points are sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.
The order of the exercises does not correspond to their difficulty level.

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1. Given jobs $J_1, \cdots, J_8$ with processing time P_j^1 on machine 1 and processing time P_j^2 on machine 2 according to the following table:

Job	P_j^1	P_j^2
1	3	2
2	2	4
3	0	7
4	6	5
5	1	2
6	4	3
7	3	3
8	5	0

The jobs will be processed on both machines, first on machine one and then in the same order on machine two. Each machine can only process one job at a time.

Consider the problem to find a schedule that minimize the makespan

$$C_{\max} = \max_{k=1,\cdots,8} \{C_k\},$$

where C_k is the completion time of job k, and where we assume that the jobs are started at time zero.

(a) First describe the rule that determine which order the jobs should be scheduled in, then apply it on this example. ....(8p)

(b) Determine the completion times for all 8 jobs. ....(4p)

2. (a) Assume that we have suppliers of a product at locations $(x_S^{(j)}, y_S^{(j)})$ with capacity C_j tons per year each, where $j = 1, \cdots, m$. The demand of this product is located in another region at locations $(x_D^{(i)}, y_D^{(i)})$ with demand D_i tons per year each where $i = 1, \cdots, n$. Assume that the total capacity is equal to the total demand.

Two cross-docks are now going to be built. One will be used by the suppliers to fill up the trucks going to the second cross-dock, where the loads will be divided up to be distributed on separate trucks to each demand location.

On average the trucks leaving the suppliers are half full and the trucks going to the suppliers are only filled to 30%.

Make suitable assumptions to solve the problem where to place the two cross-docks, and determine an optimization formulation. .... (5p)

(b) Could you recommend other alternative distribution systems for the product? What would be the advantages and drawbacks? .... (3p)

3. Consider the procurement of the maintenance of a cold-freight service company from a life cycle cost point of view. Assume that the time horizon is 1000 days. To simplify things we consider only one depot from where the trucks originate and there is the possiblity to have a workshop. Each truck has a trailer with cooling capabilities.

There are two competing offers that should be evaluated.

Alternative 1, there is a possibility to keep an inventory of trailers at the depot, and repair of trailers is done by sending the trailer to an external repair shop who will return a working trailer in 10 days. There is a trailer failure with intensity $\lambda = 0.8$ [trailers per day].

Alternative 2, there is a possibility to keep spare parts for the trailers as inventories at the depot workshop. Each trailer have three replaceable parts, which fail with intensities $\lambda_1 = 0.4$, $\lambda_2 = 0.3$ and $\lambda_3 = 0.1$ [units per day]. The defect parts are repaired by sending it to the external repair shop who will return a working unit in 10 days. The extraction of the trailer parts and the mounting of the parts is assumed to be instantaneous, but require an extra cost of 20.000 SEK per day for renting facilities and the competence.
Note that in alternative 2 there will not be any spare trailers, only spare replaceable parts, and it is assumed that all parts need to be working in order for the trailer to work.

Assume that the cost per trailer repair for alternative 1 is 30.000 SEK, and 10.000 SEK the cost per unit repair for alternative 2.

Assume that the investment cost for an trailer is 1 miljon SEK, and for the units are 500.000, 200.000, 300.000 SEK.

Compare the (minimal) life cycle costs of the two alternatives if they are both required to have a total expected number of backorders less or equal to 4.

If there is a fee for backorders that is 1.000 SEK per unit and day, can it be economically motivated to increase the inventory level for the two alternatives?

λT	1	2	3	4	5	6	7	8	9	10
k=0	0.63212	0.86466	0.95021	0.98168	0.99326	0.99752	0.99909	0.99966	0.99988	0.99995
k=1	0.26424	0.59399	0.80085	0.90842	0.95957	0.98265	0.9927	0.99698	0.99877	0.9995
k=2	0.080301	0.32332	0.57681	0.7619	0.87535	0.93803	0.97036	0.98625	0.99377	0.99723
k=3	0.018988	0.14288	0.35277	0.56653	0.73497	0.8488	0.91823	0.95762	0.97877	0.98966
k=4	0.0036598	0.052653	0.18474	0.37116	0.55951	0.71494	0.82701	0.90037	0.94504	0.97075
k=5	0.00059418	0.016564	0.083918	0.21487	0.38404	0.55432	0.69929	0.80876	0.88431	0.93291
k=6	0.000083241	0.0045338	0.033509	0.11067	0.23782	0.3937	0.55029	0.68663	0.79322	0.86986
k=7	0.000010249	0.0010967	0.011905	0.051134	0.13337	0.25602	0.40129	0.54704	0.6761	0.77978
k=8	$1.1252 \cdot 10^{-6}$	0.00023745	0.003803	0.021363	0.068094	0.15276	0.27091	0.40745	0.54435	0.66718
k=9	$1.1143 \cdot 10^{-7}$	0.000046498	0.0011025	0.0081322	0.031828	0.083924	0.1695	0.28338	0.41259	0.54207
k=10	$1.0048 \cdot 10^{-8}$	$8.3082 \cdot 10^{-6}$	0.00029234	0.0028398	0.013695	0.042621	0.098521	0.18411	0.29401	0.41696

Table 1: Tabulated values of $R(k)$ for different values of λT.

The recursive formula for determining EBO is given by

$$EBO(s+1) = EBO(s) - R(s), \quad EBO(0) = \lambda T,$$

where $R(s)$ is the probability of shortage given an initial inventory level of s.
. . . . (10p)

4. Consider a warehouse that is the sole supplier of a retailer of a certain product. Assume that neither the warehouse or the retailer allows shortage. The demand of the product that the retailer see is constant and deterministic with a rate of 1000 units per week. The inventory cost is 100 SEK per unit and week at the retailer and 10 SEK per unit and week at the warehouse. The ordering cost is 500 SEK per order at the retailer and 1000 SEK per order at the warehouse.

(a) How much would the warehouse and retailer order each time that they order, and how often would they order, if the retailer and warehouse do not cooperate.

We assume that the warehouse knows the order plan of the retailer so that he can synchronize his orders. . . . . (3p)

(b) If they cooperate what would the optimal order quantities be? and how often would they order? . . . . (5p)

(c) Assume now that you are a manufacturer of the product and you should deliver the products to the warehouse. You want to determine the production capacity, number of units manufactured per week, for your factory.

How would you do that? Which factors do you think are important, how would you set up your production plan?

Consider the case described here and other more realistic aspects and extensions. . . . . (5p)

5. Consider a shop that sells a product which has a demand that is such that the time between two purchases is exponentially distributed with mean time T. Each time the shopowner orders new products he has to pay an order cost which is K dollars, and the delivery time is t time units. There is also a holding cost of h dollars per unit and time unit.

We want to design a (s, S) ordering strategy.

We want to determine the value of s so that the probability of shortage is less than 5%, but to avoid too much computations it is enough to determine an inequality that defines s, that may contain known (and defined) probability functions.

We want to determine the value of S so that the cost per time unit is minimized. In order to do that determine an expression for the expected cost for an ordering cycle. .... (7p)

Good luck!