KTH Mathematics

Exam in SF2866 Applied Systems Engineering, SF2868 Systems engineering, Business and Management, Part 1 Monday October 24 , 2016, 14.00–19.00

Examiner: Per Enqvist, tel. 790 62 98

Allowed tools: Pen/pencil, ruler and eraser. A formula sheet is handed out.
No calculator is allowed!

Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Explain carefully the methods you use, and always define the variables and notation you use. Conclusions should always be motivated.

Note! Personal number must be written on the title page. Write only one exercise per sheet. Number the pages and write your name on each sheet!
25 points are sufficient for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.
The order of the exercises does not correspond to their difficulty level.

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1. Given a consultancy firm that has projects P_1, P_2, P_3 to finish. The projects can not be done in parallell. As each project is finished a premium of c_1, c_2, c_3 thousands of dollars is collected. The time to finish each project is t_1, t_2, t_3 weeks respectively.

Now, the firm wants to determine the order to do the projects so that the total premium plus interest is maximized.

The interest rate is ϵ% per week.

Show that this is equivalent to minimizing the total weighted completion time for some choice of weights w_j if we replace the factors $(1+\epsilon/100)^t$ with the first order approximations $1+(\epsilon/100)t$.

Let

j	c_j	t_j
1	6	8
2	4	2
3	5	5

and $\epsilon = 0.5$. Determine the optimal order to do the jobs. Make sure you explain how the order of the jobs is determined. .... (10p)

2. *A cross-dock is a reloading facility where at one end incoming trucks are unloaded and at the other end outgoing trucks are loaded. The purpose is to sort goods intended for different destinations and/or combine goods from different suppliers to increase the efficiency of the transportation system.*

(a) Assume that there is a cross-dock with m reception-docks where trucks come to unload goods, and n delivery-docks where trucks come to load the goods.

Assume that the arrival times of trucks to the reception-docks are given by t_1 to t_M and the arrival times of trucks to the delivery-docks are given by s_1 to s_N. Unloading and loading a truck takes 30 minutes.

Use multi-commodity network flow formulation to model the problem, where the objective is to minimize the number of used docks. ....(5p)

(b) Assume that $W_{k\ell}$ is the weight of the goods arriving on truck k to be transported to truck ℓ, and let d_{ij} be the distance from reception-dock i to delivery dock j. The weight $W_{k\ell} > 0$ only if there is enough time between the arrival of truck k and departure of truck ℓ to unload-move-load the goods.

What would the objective function be if we want to minimize the total transported mass times distance moved at the cross dock. ....(3p)

(c) Discuss possible extensions that could be relevant to model a real cross-dock. (2p)

3. Assume three pizzerias with one delivery car each are supposed to deliver orders to N customers. Let (x_i, y_i) be the location of the pizzerias and (z_i, w_i) be the location of the customers.

Formulate an optimization problem minimizing the total driving distance of the delivery cars.

Assume that the orders for next two hours were given above, and after one hour of delivieries there are M new orders that have come in. How would you plan the deliveries of the new orders?

What are the main differences between this problem and the home care service problem considered in project 2? .... (10p)

4. Consider a warehouse that is the sole supplier of a retailer of a certain product. Assume that neither the warehouse or the retailer allows shortage. The demand of the product that the retailer see is constant and deterministic with a rate of 2000 units per week. The inventory cost is 200 SEK per unit and week at the retailer and 100 SEK per unit and week at the warehouse. The ordering cost is 1000 SEK per order at the retailer and 2000 SEK per order at the warehouse.

(a) How much would the warehouse and retailer order each time that they order, and how often would they order, if the retailer and warehouse do not cooperate. We assume that the warehouse knows the order plan of the retailer so that he can synchronize his orders. ....(3p)

(b) If they cooperate what would the optimal order quantities be? and how often would they order? .... (5p)

(c) Assume that one more level is added to the supply chain. What kind of complications would you expect?
.... (2p)

5. Consider an aircraft routing problem, for six flights between locations A, B and C.
Flight 1 departs A at 8.00 and arrives to B at 17.00.
Flight 2 departs B at 10.00 and arrives to A at 19.00.
Flight 3 departs B at 12.00 and arrives to C at 15.00.
Flight 4 departs C at 16.00 and arrives to B at 19.00.
Flight 5 departs C at 16.00 and arrives to A at 23.00.
Flight 6 departs A at 8.00 and arrives to C at 15.00.

Determine all legal pairings, i.e., satisfying the following constraints:
Each pairing starts and ends in A.
Each duty period is less than 12 hours.
There is at most one layover (at most two duty periods).

Determine a set covering/partitioning formulation (linear integer program) of the problem to minimize the number of used aircrafts needed to fly all the flights.

What is the minimal number of aircrafts in this case?
Are the optimal pairings unique?
and can you find an optimal choice of pairings? .... (10p)

Good luck!