Exercise 13.8

We have $f'(x) = 2xe^{x^2}$ and so $f'(0) = 0$.

Moreover $f''(x) = 2e^{x^2} + 2x \cdot 2xe^{x^2}$

$$= 2e^{x^2} + 4x^2e^{x^2} = 2(1+2x^2)e^{x^2}$$

And so $f''(0) = 2(1)e^0 = 2 > 0$.

Exercise 13.9

We have $g'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$

and so $g'(x) = 0$ iff $x \in \{0, 1\}$.

Also, $g''(x) = 24x(x-1) + 12x^2 = 12x\,[2x - 2 + x] = 12x\,[3x-2]$.

Thus $g''(0) = 0$, and $g''(1) = 12 > 0$.

So 1 is a local minimizer.

We note that $g(x) = x^3(3x-4) + 1 = x^3(3x-4) + g(0)$.

Thus $g(x) - g(0) = x^3(3x-4)$.

Since for $0 < x < \frac{4}{3}$ we have $x^3(3x-4) < 0$, $g(x) < g(0)$ for all such x. On the other hand for all $x < 0$, $3x - 4 < 0$ and $x^3 < 0$, so that $x^3(3x-4) > 0$ and so $g(x) > g(0)$.

So 0 is not a local minimizer.