FORMULA-SHEET FOR SF1851

Simplex method for linear programming problems in standard form:

$$A_\beta \overline{b} = b,$$
$$A_\beta^\top y = c_\beta,$$
$$r_\nu = c_\nu - A_\nu^\top y.$$

Stop if $r_\nu \geq 0$. Otherwise take q such that r_{ν_q} is the most negative component of r_ν.

$$A_\beta \overline{a}_{\nu_q} = a_{\nu_q}.$$

Stop if $\overline{a}_{\nu_q} \leq 0$. Otherwise find p so that $t_{\max} = \min\left\{ \dfrac{\overline{b}_k}{\overline{a}_{\nu_q,k}} : \overline{a}_{\nu_q,k} > 0 \right\} = \dfrac{\overline{b}_p}{\overline{a}_{\nu_q,p}}$.

New basic tuple is taken as $\beta = (\beta_1, \ldots, \beta_{p-1}, \nu_q, \beta_{p+1}, \ldots, \beta_m)$.

Primal problem and its dual problem (in general form):

$$\begin{array}{rlrl} \text{minimize} & c_1^\top x_1 + c_2^\top x_2 & \text{maximize} & b_1^\top y_1 + b_2^\top y_2 \\ \text{subject to} & A_{11}x_1 + A_{12}x_2 \geq b_1, & \text{subject to} & A_{11}^\top y_1 + A_{21}^\top y_2 \leq c_1, \\ & A_{21}x_1 + A_{22}x_2 = b_2, & & A_{12}^\top y_1 + A_{22}^\top y_2 = c_2, \\ & x_1 \geq 0, \quad x_2 \text{ free.} & & y_1 \geq 0, \quad y_2 \text{ free.} \end{array}$$

Network flow problem:

(Simplex multipliers) $y_i - y_j = c_{ij}$ for tree edges (i, j).

(Reduced costs for nonbasic variables) $r_{ij} = c_{ij} - (y_i - y_j)$ for nontree edges (i, j).

Quadratic optimization problem $\left\{ \begin{array}{ll} \text{minimize} & \frac{1}{2}x^\top Hx + c^\top x + c_0 \\ \text{subject to} & x \in \mathbb{R}^n \end{array} \right\}$: $H\widehat{x} = -c$

Quadratic optimization problem $\left\{ \begin{array}{ll} \text{minimize} & \frac{1}{2}x^\top Hx + c^\top x + c_0 \\ \text{subject to} & Ax = b \end{array} \right\}$:

(Nullspace method) $A\overline{x} = b$; $\widehat{x} = \overline{x} + Zv$ and $(Z^\top HZ)v = -Z^\top(H\overline{x} + c)$;

(Lagrangian method) $H\widehat{x} - A^\top u = -c$ and $A\widehat{x} = b$.

Least squares problem $\left\{ \begin{array}{ll} \text{minimize} & \frac{1}{2}\|Ax - b\|^2 \\ \text{subject to} & x \in \mathbb{R}^n \end{array} \right\}$:

(A has independent columns) $A^\top A\overline{x} = A^\top b$;

(A has dependent columns) $AA^\top \widehat{u} = A\overline{x}$ and $\widehat{x} = A^\top \widehat{u}$.

Newton's method for $\left\{\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in \mathbb{R}^n \end{array}\right\}$: $F(x^{(k)})(x^{(k+1)} - x^{(k)}) = -(\nabla f(x^{(k)}))^\top$.

Newton-Gauss method for nonlinear least squares problem $\left\{\begin{array}{ll} \text{minimize} & \frac{1}{2}\|h(x)\|^2 \\ \text{subject to} & x \in \mathbb{R}^n \end{array}\right\}$:

$(\nabla h(x^{(k)}))^\top \nabla h(x^{(k)})(x^{(k+1)} - x^{(k)}) = -(\nabla h(x^{(k)}))^\top h(x^{(k)})$.

Lagrange's method for $\left\{\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & h(x) = 0 \end{array}\right\}$: $h(x_0) = 0$, $\nabla f(x_0) + u^\top \nabla h(x_0) = 0$.

KKT-conditions for $\left\{\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & g(x) \leq 0 \end{array}\right\}$:

(KKT-1) $\nabla f(x_0) + y^\top \nabla g(x_0) = 0$
(KKT-2) $g(x_0) \leq 0$
(KKT-3) $y \geq 0$
(KKT-4) $y^\top g(x_0) = 0$.

End of the formula sheet.