KTH Matematik

SF2842: Geometric Control Theory

Solution to Homework 1

Due February 7, 16:59, 2017

You may discuss the problems in group (maximal **two** students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. Consider the system

$$\begin{aligned} \dot{x} &= Ax + Bu = \begin{pmatrix} -2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} u \\ y &= Cx = (1\ 0\ 0\ 0)x. \end{aligned}$$

(a) Is the system observable? . . . . (1p)

Answer: Yes

(b) Compute $\mathcal{V}^*$ and $\mathcal{R}^*$ contained in $\mathcal{V}^*$, and find (parameterize) ALL friends F of $\mathcal{V}^*$. . . . . (3p)

Answer: $V^* = \{x : x_1 = 0, x_3 = 0\}, R^*\{x : x_1 = x_3 = x_4 = 0\}$.

(c) Let $A_F = A + BF$, and $\Omega_F = (C^T, A_F^T C^T, \cdots, (A_F^3)^T C^T)^T$, find an F that maximizes the dimension of $ker\ \Omega_F$ and $A + BF$ is a stable matrix. . . . . . . (2p)

Answer: F should be a friend of V^* and there are many choices.

2. Consider the system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned}$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$. Determine if each of the following statements is true or false. You must justify your answer!

(a) If the dimension of V^* is greater than 0, then for any friend F of V^*, $(C, A+BF)$ is not observable. . . . . (2p)

Answer: True, since $Cx(t) = 0$ if $x_0 \in V^*$.

(b) Suppose the system is controllable, then any controlled invariant subspace is also a controllability subspace. . . . . (2p)

Answer: False. Counter example: $x_1 - x_2 = 0$ for $\dot{x}_1 = x_2, \dot{x}_2 = -x_1 + u$.

3. Consider a SISO

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned}$$

where $g(s) = C(sI - A)^{-1}B = \frac{s^m + p_1 s^{m-1} + \cdots + p_m}{s^n + d_1 s^{n-1} + \cdots + d_n}$
Use Ω^* algorithm to show that the dimension of V^* is m. . . . . (3p)

Answer: By relative degree $r = n - m$, we have $cA^i \in G^\perp,\ i = 0, \cdots, r-2$. Thus $\Omega_{r-1} = span\{c, cA, \cdots, cA^{r-1}\}$. Since $cA^{r-1}b \neq 0$, $cA^{r-1} \notin G^\perp$, which implies $\Omega_{r-1} \cap G^\perp = \Omega_{r-2}$, thus $\Omega_r = \Omega_{r-1}$. $dim\ V^* = n - dim\ \Omega_r = m$.

4. Consider

$$\begin{aligned} \dot{x}_1 &= x_1 + x_3 + u_1 \\ \dot{x}_2 &= x_2 - x_3 + u_1 \\ \dot{x}_3 &= -x_3 + 2x_4 + u_2 \\ \dot{x}_4 &= x_1 + \alpha x_2 + x_4 + u_1 \\ y_1 &= x_1 - x_2 \\ y_2 &= x_4, \end{aligned}$$

where α is a constant.

(a) What is the relative degree for the system? . . . . (1p)
Answer: $(2, 1)$.

(b) Convert the system into the normal form and compute the zero dynamics.(3p)
Answer: Omitted.

(c) When $y(t) = 0\ \forall t \geq 0$, does it always imply $\lim_{t\to\infty} x(t) = 0$? . . . . (2p)
Answer: No, only when $\alpha > 0$