KTH Matematik

SF2842: Geometric Control Theory

Solution to Homework 2

Due February 22, 16:50pm, 2017

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1. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} x + \begin{pmatrix} \alpha & 1 \\ 2 & 1 \\ \alpha & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} x, \end{aligned}$$

where α is a real constant.

(a) For what values of α is the noninteracting control problem solvable? .....(2p)
Answer: $\alpha \neq 2$.

(b) What is the (transmission) zero(s) of the system when the noninteracting control problem solvable? .... (2p)
Answer: -1.

(c) Suppose now the first output y_1 is taken away from the system, namely only y_2 is kept. What is the (transmission) zero(s) of the system now if $\alpha = 2$? (3p)
Answer: We can regard $2u_1 + u_2$ as one control, then the system is equivalent to a SISO system. The zeros are $-1,\ -2$.

2. Consider the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -x_1 - 3x_2 - 3x_3 + w_1 \\ \dot{w}_1 &= 2w_2 \\ \dot{w}_2 &= -2w_1 \\ y &= c_1 x_1 + c_2 x_2 + x_3, \end{aligned}$$

where $c_1,\ c_2$ are real constants and $c_1 - c_2 + 1 \neq 0$.

(a) Compute the invariant subspace $x = \Pi w$. [2p]
Answer: Denote $\dot{w} = Sw$. Let the invariant subspace be $x_i = \pi_i w,\ i = 1, 2, 3$. Solving the Sylvester equation we have $\pi_2 = \pi_1 S,\ \pi_3 = \pi_1 S^2$, and $\pi_1 S^3 = -\pi_1 - 3\pi_1 S - 3\pi_1 S^2 + [1\ 0]$. Thus $\pi_1 = \frac{1}{125}[-11\ 2]$.

(b) For what value(s) of c_1, c_2 is the above system (consisting of x and w) unobservable? Explain why. [2p]

Answer: We can view the above system as a control system with $u = w_1$. There will not be zero pole cancellation if $c_1 - c_2 + 1 \neq 0$, thus the control system is both controllable and observable. The exo system is also observable. Thus the overall system is unobservable iff an eigenvalue of the exo system is a transmission zero, namely $c_1 = 4,\ c_2 = 0$.

(c) Can we find c_1, c_2 such that $y(t) = w_1(t)$ in the steady state? [2p]

Answer: This is true since $\pi_1,\ \pi_1 S$ must be linearly independent.

3. Consider a control system subject to disturbance:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -2x_1 - x_2 + x_3 + u + 2w_1 \\
\dot{x}_3 &= \alpha x_3 + u \\
y &= x_1,
\end{aligned}$$

where w_1 is an unknown nonzero constant (disturbance) and α is a real constant.

(a) Is the disturbance decoupling problem (DDP) solvable? .... (1p)

Answer: No.

(b) When $u = 0$, show that if $\alpha < 0$, then for all initial conditions, $y(t) \to w_1$ as $t \to \infty$. .... (1p)

Answer: In this case we can easily show that on the invariant subspace, $x_1 = w_1$.

(c) For what values of α is the error feedback output regulation problem guaranteed to be solvable if we choose $y_r = 0$ as the reference output (you do not need to design the controller)? .... (2p)

Answer: We need to ensure that the augmented system is observable (detectable is not required for this exercise) and 0 is not a transmission zero, which implies $\alpha \neq 0, 1$.

(d) For $\alpha = 2$, solve the the full information output regulation problem for $y_r = 0$. [3p]

Answer: In general, we can consider y_r as generated by $\dot{w}_2 = 0$. This gives $u = -k(x_3 - (2 \quad - 2)w) - (4 \quad - 4)w$, where $k > 0$. With the particular $y_r = 0$, one can set $w_2 = 0$.