KTH Matematik

5B1823: Geometric Control Theory

Homework 2

Due November 29, 16:50pm, 2006

You may discuss the problems in group (maximal two students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. Consider the system

$$\begin{aligned}\dot{x} &= \begin{pmatrix} 0 & a & 1+2a & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} u \\ y &= \begin{pmatrix} 1 & 0 & 2+a & 0 \end{pmatrix} x,\end{aligned}$$

where $a \neq 0$ is a constant.

(a) What is the zero dynamics? [1p]

(b) Use the Rosenbrock matrix to verify your computation of the transmission zero from (a). [1p]

(c) For what a we can use high gain output feedback control $u = -ky$ to stabilize the system? [1p]

2. Consider the system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -x_1 - 3x_2 - 3x_3 + w_1 \\ \dot{w}_1 &= 2w_2 \\ \dot{w}_2 &= -2w_1 \\ y &= ax_1 + x_3,\end{aligned}$$

where $a \neq 0$ is constant.

(a) Compute the invariant subspace $x = \Pi w$ (the matlab command "lyap" can be used). [1p]

(b) For what value of a is the above system (consisting of x and w) unobservable? Explain why. [2p]

3. Consider the car steering example:

$$\begin{aligned}\dot{\alpha}_f &= -2\alpha_f + r + 0.3\dot{\delta}_f\\ \dot{\psi} &= r\\ \dot{r} &= -0.6\alpha_f - 2\psi + \delta_f + d(t),\end{aligned}$$

where the driver's goal is to keep the orientation straight ($\delta_f = -0.8\psi$), $d(t)$ is a sinusoidal disturbance $a\sin(4t+\theta)$ with unknown amplitude and phase.

Design an output that is a linear combination of ψ and r, such that the output optimally reconstructs the disturbance in stationarity. [2p]

4. Consider:

$$\begin{aligned}\dot{x}_1 &= x_2 + 2x_4\\ \dot{x}_2 &= x_2 + u_1\\ \dot{x}_3 &= -2x_3 + w_3 + u_2\\ \dot{x}_4 &= x_1 - x_3 - x_4 + u_2\\ \dot{w}_1 &= w_2\\ \dot{w}_2 &= -w_1\\ \dot{w}_3 &= 0\\ e_1 &= x_1 - w_1\\ e_2 &= x_4 - w_2\end{aligned}$$

(a) Find a control $u = Kx + Ew$ that solves the full information output regulation problem. [2p]

(b) In the closed-loop system, for $x(0) = 0$ and $w_1(0) = 0,\ w_2(0) = 1,\ w_3(0) = 1$, plot x_1 vs x_4 for a while in Matlab until you see the pattern for the stationary response. What does it look like in stationarity? [1p]

(c) Is the error feedback output regulation solvable in this case? [1p]