KTH Matematik

5B1822: Geometric Systems Theory

Homework 1

Due November 16, 16:50pm, 2004

You may discuss the problems in group (maximal three students in a group), but each of you **must** write and submit your own report. Write the names of persons that you cooperated with.

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1. [2p]. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 2 & 3 & -2 & -1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 & -1 \\ 0 & 1 \\ 1 & 0 \\ -1 & 0 \end{pmatrix} u \\ y &= (1\ 1\ 0\ 0)x. \end{aligned}$$

Compute $\mathcal{V}^*$ and find all friends F of $\mathcal{V}^*$.

2. [2p]. Consider the same system as in Problem 1.

(a) Given $x_1 = (0,0,0,0)^T$ and $x_2 = (0,0,1,-1)^T$, can we find a control $u = Fx + Gv$ such that in some finite time T, $x(T, x_1) = x_2$ while $y(t)$ is kept at 0 (i.e. $y(t) = 0$ for all $0 \le t \le T$)? If the answer is yes, please give such an F and G. (Here, $x(t, x_1)$ is the solution of $\dot{x} = Ax + Bu$, $x(0) = x_1$.)

(b) If we replace x_2 by $(-1,1,0,-2)^T$, can we also find such a control?

3. [3p]. Consider

$$\dot{x} = \begin{pmatrix} -2 & 1 & 0 \\ 0 & -2 & 2 \\ 1 & 2 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} u + Ew,$$

where w is the disturbance.

(a) For $y = (1\ 1\ 0)x$, derive the minimum constraint on E such that DDP is solvable. Find a state feedback $u = Fx + v$ which solves the DDP problem.

(b) For any $u = Fx + v$ that solves the DDP problem for the above y, what is the dimension of the unobservable subspace for the corresponding closed-loop system? On the other hand, is the system observable when $u = 0$?

(c) Can we find an output feedback $u = Ky + v$ that solves the DDP ?

4. [4p]. Consider

$$\begin{aligned}
\dot{x}_1 &= -x_1 + x_2 \\
\dot{x}_2 &= -x_2 + 2x_3 + u_1 \\
\dot{x}_3 &= x_1 - x_4 \\
\dot{x}_4 &= x_2 - 3x_4 - u_1 + u_2 \\
y_1 &= x_1 + x_2 \\
y_2 &= x_3
\end{aligned}$$

(a) What is the relative degree for the system?

(b) Convert the system into the normal form and compute the zero dynamics.

(c) What is $\mathcal{V}^*$?

(d) What is $\mathcal{R}^*$ contained in ker C?