KTH Matematik

5B1822: Geometric Systems Theory

Homework 1

Due November 18, 16:50pm, 2005

You may discuss the problems in group (maximal **two** students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. [3p]. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 1 & 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} u \\ y &= (0\ 1\ 1\ 0)x. \end{aligned}$$

(a) Compute $\mathcal{V}^*$ and find all friends F of $\mathcal{V}^*$.

(b) Computer $\mathcal{R}^*$ that is contained in $ker\ C$.

2. [2p]. Consider the same system as in Problem 1.

(a) Given $x_1 = (0,0,0,0)^T$ and $x_2 = (1,0,0,0)^T$, show that for **any** given finite time T, we can find a control $u = Fx + Gv$ such that $x(T, x_1) = x_2$ while the trajectory of $x(t, x_1)$ is a straight line. (Here, $x(t, x_1)$ is the solution of $\dot{x} = Ax + Bu$, $x(0) = x_1$.)

(b) If we let $x_1 = (0, 1, -1, 0)^T$, and $x_2 = (1, k, -k, 0)^T$, for what values of k x_2 can be reached in **some** finite time from x_1?

3. [2p]. Consider

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u + Ew, \\ y &= (-1\ 0\ 1)x \end{aligned}$$

where w is the disturbance.

(a) Derive the minimum constraint on E such that DDP is solvable. Find a state feedback $u = Fx + v$ that solves the DDP problem.

(b) Can we find a $u = Fx + v$ that solves the DDP problem for any E that meets the minimum constraint obtained above while makes the closed-loop system stable, i.e. $A + BF$ has only eigenvalues with negative real part?

4. [5p]. Consider

$$
\begin{aligned}
\dot{x}_1 &= -x_1 + x_4 + u_1 \\
\dot{x}_2 &= x_1 \\
\dot{x}_3 &= x_2 + x_4 + u_2 \\
\dot{x}_4 &= x_2 + u_2 \\
y_1 &= x_1 + x_2 \\
y_2 &= x_3 - x_4
\end{aligned}
$$

(a) What is the relative degree for the system?

(b) Convert the system into the normal form and compute the zero dynamics.

(c) Is it possible to find a control such that the system can be steered from the initial state $x_0 = (1\ -1\ 0\ 0)^T$ to $x_1 = (2\ -2\ 0\ 0)^T$ at some finite time T while the trajectory that connects x_0 and x_1 is a straight line?

(d) Is it possible at all to find a control such that the system can be steered from the initial state $x_0 = (1\ -1\ 0\ 0)^T$ to $x_1 = (2\ -2\ 0\ 0)^T$ at some finite time T (no particular trajectory is required) ?