5B1822: Geometric Systems Theory
Homework 2
Due December 2, 16:50pm, 2005

You may discuss the problems in group (maximal two students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. Consider the system

$$\dot{x} = \begin{pmatrix} -1 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} x + \begin{pmatrix} -1 & -1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x.$$

   (a) What is the zero dynamics? [1p]

   (b) Use the Rosenbrock matrix to verify your computation of the transmission zero from (a). [1p]

   (c) Solve the noninteracting control problem. [1p]

2. Consider the system

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -2x_1 - 3x_2 + w_1 \\
\dot{w}_1 &= w_1 + w_2 \\
\dot{w}_2 &= 2w_2 \\
y &= -x_1 + x_2
\end{aligned}$$

   (a) Compute the invariant subspace $x = \Pi w$ (the matlab command "lyap" can be used). [1p]

   (b) Is the above system (consisting of x and w) observable or not? Why so? [1p]

3. Consider the car steering example:

$$\begin{aligned}
\dot{\alpha}_f &= -2\alpha_f + r + \dot{\delta}_f \\
\dot{\psi} &= r \\
\dot{r} &= -0.6\alpha_f - 2\psi + 3\delta_f + d(t),
\end{aligned}$$

   where the driver's goal is to keep the orientation straight ($\delta_f = -0.8\psi$), $d(t)$ is a sinusoidal disturbance $a \sin(2t + \theta)$ with unknown amplitude and phase.

   Design an output that is a linear combination of ψ and r, such that the output optimally reconstructs the disturbance in stationarity. [2p]

4. Consider:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_2 + x_3 + u_1$$
$$\dot{x}_3 = w_3 + u_2$$
$$\dot{x}_4 = x_1 - \alpha x_3 - x_4 + u_2$$
$$\dot{w}_1 = w_2$$
$$\dot{w}_2 = -w_1$$
$$\dot{w}_3 = 0$$
$$e_1 = x_1 - w_1$$
$$e_2 = x_4 - w_2$$

(a) For $\alpha = 1$, find a control $u = Kx + Ew$ that solves the full information output regulation problem. [2p]

(b) In the closed-loop system, for $x(0) = 0$ and $w_1(0) = 0$, $w_2(0) = 1$, $w_3(0) = 1$, plot x_1 vs x_4 for a while in Matlab until you see the pattern for the stationary response. What does it look like in stationarity? [1p]

(c) Is the error feedback output regulation solvable in this case? [1p]

(d) What are the real value(s) of α such that the regulation problem may not be solvable? [1p]