SF2842: Geometric Control Theory, 2012
Answers of Homework 1

November 20, 2012

1.

(a) $V^* = \text{span}\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\} = \left\{ x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} | x_1 + x_2 = 0, x_4 = 0 \right\}.$

The friend of V^* has the form

$$\begin{pmatrix} * & * & * & * \\ \alpha & \alpha+1 & -1 & * \end{pmatrix}.$$

(b) $R^* = \text{span}\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\} = V^*.$

2.

(a) The controllable subspace is $\Gamma =< A|\text{Im}B >$. For any matrix $F \in R^{m\times n}$ and any $x \in \Gamma$, we have $(A + BF)x = Ax + BFx \in \Gamma + \text{Im}B = \Gamma$ since Γ is A-invariant and $\text{Im}B \subseteq \Gamma$. Hence, Γ is $(A + BF)$-invariant for any F.

(b) It is straightforward that the unobservable subspace $\Omega = \ker \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{pmatrix}$ is A-invariant.

But Ω is not $(A + BF)$=invariant for all F. Counter example can be

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \; C = \begin{pmatrix} 1 & 0 \end{pmatrix} \text{ and } F = \begin{pmatrix} 0 & 1 \end{pmatrix}.$$

(c) If the unobservable subspace $\Omega = \{0\}$, then it will be $(A + BF)$-invariant for all F. If Ω contains some non-zero element, then it cannot be $(A+BF)$-invariant for all F while the system has a relative degree. We know that $\Omega = \{0\}$ if and only if $\sum_{i=1}^{m} r_i = n$.

3.

(a) $V^* = V^* = \text{span}\left\{ \begin{pmatrix} c \\ -1 \\ \frac{1}{c} \end{pmatrix} \right\}$. DDP is solvable if and only if $\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} \in V^*$.

(b) The zero dynamics of the system will be $\dot{z} = -\frac{1}{c}z$. So we just need $c > 0$.

4.
(a) When $a \neq -1$, the system will have a relative degree (1,2).
(b) The zero dynamics is $\dot{z} = \frac{1-a}{1+a} z$.