KTH Matematik

SF2842: Geometric Control Theory

Solution to Homework 2

Due November 27, 16:50pm, 2008

You may discuss the problems in group (maximal two students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

---

1. Consider the system

$$
\begin{aligned}
\dot{x} &= \begin{pmatrix} -1 & 0 & -1 & -2 \\ 0 & 0 & 1 & 2 \\ -1 & -1 & 2 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} x + \begin{pmatrix} -1 & -1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} u \\
y &= \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix} x.
\end{aligned}
$$

(a) What is the zero dynamics? [1p]

$\dot{z} = -z$.

(b) Use the Rosenbrock matrix to verify your computation of the transmission zero from (a). [1p]

(c) Solve the noninteracting control problem. [1p]

Since the system has rel. degree $(2, 1)$, the noninteracting problem is solvable. Let $z = x_4, \xi_1^1 = x_1 + x_2, \xi_2^1 = -x_2, \xi_1^2 = 2x_3$.

2. Consider the system

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_1 - 2x_2 + w_1 \\
\dot{w}_1 &= w_1 + w_2 \\
\dot{w}_2 &= w_2 \\
y &= x_1 - x_2
\end{aligned}
$$

(a) Compute the invariant subspace $x = \Pi w$ if it exists (the Matlab command "lyap" can be used). [2p]

(b) Is the above system (consisting of x and w) observable or not? Why so? [1p]

Not observable, since s=1 is a transmission zero.

3. Consider the car steering example:

$$\dot{\alpha}_f = -2\alpha_f + r + \dot{\delta}_f$$
$$\dot{\psi} = r$$
$$\dot{r} = -0.6\alpha_f - 2\psi + 3\delta_f + d(t),$$

where the driver's goal is to keep the orientation straight, i.e., $\delta_f = -0.5\psi$, and $d(t)$ is a sinusoidal disturbance $a\sin(2t+\theta)$ with unknown amplitude and phase.

Design an output that is a linear combination of ψ and r, such that the output optimally reconstructs the disturbance in stationarity. You may use Matlab for computation. [3p]

4. Consider:

$$\begin{aligned}
\dot{x}_1 &= \alpha x_1 - x_4 + w_3 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= u_2 \\
\dot{x}_4 &= -x_3 - x_4 - u_1 + u_2 \\
\dot{w}_1 &= w_2 \\
\dot{w}_2 &= -w_1 \\
\dot{w}_3 &= w_1 \\
e_1 &= x_2 - w_2 \\
e_2 &= x_4 - w_1
\end{aligned}$$

(a) For $\alpha = 1$, find a control $u = Kx + Ew$ that solves the full information output regulation problem. [2p]

1. Solve the sylvester equation: $x_2 = w_2, x_4 = w_1, x_3 = -w_1, x_1 = \pi_1 w$, $u_2 = c_2 w = -w_2, u_1 = c_1 w = u_2 - x_3 - x_4 = -w_2$.

2. Let $f_2 x = -k_1 x_2 - k_2 x_3, f_1 x = -(p_1 + p_2 + 1)x_1 + p_2 x_4$, **where all coefficients are positive.**

(b) What is the real value(s) of α such that the regulation problem may not be solvable? [1p]

$\alpha = 0$.