SF2842: Geometric Control Theory, 2012
Answers of Homework 2

December 4, 2012

1.
(a) If $a = 0$, then $V^* = \text{span}\{e_2.e_3, e_4\}$, and $V^* \cap \text{Im}B = \text{Im}B \neq 0$.
If $a \neq 0$, then the relative degree of the system is 2. Then the system is invertible.
Therefore, the system is invertible iff $\underline{a \neq 0}$.

(b) If $a \neq 0$, then the transmission zeros s should satisfy the equation

$$s^2 + s + 2a = 0,$$

which gives $\underline{s_{1,2} = -\frac{-1 \pm \sqrt{1-8a}}{2}}$.

If $a = 0$, then $R^* = V^*$, and the number of transmission zeros are $\dim V^* - \dim R^* = 0$. <u>There is no transmission zero</u>.

(c) High gain control is achievable when the zero dynamics is stable $\Leftrightarrow \underline{a > 0}$.

2.
In this problem, $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $P = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\Gamma = \begin{pmatrix} 0 & \beta \\ -\beta & 0 \end{pmatrix}$, $c = \begin{pmatrix} -\alpha & 0 & 1 \end{pmatrix}$ and $q = \begin{pmatrix} 1 & 0 \end{pmatrix}$.

(a) The solution to the Sylvester equation $A\Pi - \Pi\Gamma = -Pq$ is

$$\underline{\Pi = -\frac{1}{4} \begin{pmatrix} -1 & -1 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}}$$

(b) In order to have the whole system observable, we need 1) the pair (c, A) to be observable, 2) the pair (q, Γ) to be observable and 3) eigenvalues of Γ are not the transmission zero of the system

$$\begin{aligned} \dot{x} &= Ax + Pu \\ y &= cx. \end{aligned}$$

1) $\Rightarrow \alpha \neq 1$

2) always holds

3) $\Rightarrow \alpha \neq -1$

Therefore, when $\underline{\alpha \neq \pm 1}$, the whole system is observable.

(c) The FIORP is solvable when the eigenvalues of Γ are not the transmission zero of the system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= cx. \end{aligned}$$

The only case when this happens is when $\alpha = -3$ and $\beta = \sqrt{11}$. When $\underline{\alpha \neq -3 \text{ or } \beta \neq \sqrt{11}}$, the FIORP is solvable.

3.

The system could be rewrite as

$$\begin{aligned} \begin{pmatrix} \dot{\alpha}_f \\ \dot{\varphi} \\ \dot{r} \end{pmatrix} &= \begin{pmatrix} -2 & 0 & 0.76 \\ 0 & 0 & 1 \\ -0.6 & -2.8 & 0 \end{pmatrix} \begin{pmatrix} \alpha_f \\ \varphi \\ r \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u = A \begin{pmatrix} \alpha_f \\ \varphi \\ r \end{pmatrix} + bu, \\ \begin{pmatrix} \dot{w}_1 \\ \dot{w}_2 \end{pmatrix} &= \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \Gamma \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, \\ u = \begin{pmatrix} 1 & 0 \end{pmatrix} &\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = q \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \end{aligned}$$

The OTIP is solved by Π and c that is the solution to the equations

$$\begin{aligned} A\Pi - \Pi\Gamma &= -bq \\ c\Pi &= q, \end{aligned}$$

which gives the result

$$\underline{\Pi = \begin{pmatrix} -0.28 & -0.46 \\ -0.96 & -0.23 \\ 0.46 & -1.95 \end{pmatrix}, \ c = \begin{pmatrix} 0 & -0.97 & 0.11 \end{pmatrix}}$$