KTH Matematik

SF2842: Geometric Control Theory

Solution to Homework 3

Due December 15, 16:50pm, 2008

You may discuss the problems in group (maximal two students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. Consider the system

$$\dot{x} = g_1 u_1 + g_2 u_2,$$

where

$$g_1 = \begin{pmatrix} \cos(x_3 + x_4) \\ \sin(x_3 + x_4) \\ \sin(x_4) \\ 0 \end{pmatrix} \quad g_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

One can view this as a more complex vehicle steering system. Define:

$$Drive = g_1, \quad Steer = g_2, \quad Wriggle = [Steer, Drive], \quad Slide = \begin{pmatrix} -\sin(x_3) \\ \cos(x_3) \\ 0 \\ 0 \end{pmatrix},$$

where $[\cdot,\cdot]$ is the Lie Bracket.

- What is $[Steer, Wriggle]$ and $[Wriggle, Drive]$? [1p]
- Is the distribution $span\{g_1, g_2\}$ involutive? [1p]
- Show that the system is locally strongly accessible and controllable. [1p]

Solution: omitted.

2. Determine if each of the following statements is true or false.

- Consider the consensus control problem in R^2 with four agents: $x_i = u_i,\ x_i \in R^2,\ u_i \in R^2$. If the initial positions of the four agents form a rectangle (with the position of each agent as a vertex), then as the consensus control is applied, no agent can ever move outside the rectangle. [2p]

  Solution: True since all agents can only move inside the convex hull of the initial conditions.

- If a nonlinear control system is exactly linearizable as defined in the compendium, then it is exponentially stabilizable by feedback control. [2p]

  Solution: True since the exactly linearized system is controllable by definition.

3. Consider

$$\begin{aligned}\dot{x}_1 &= \alpha x_1 + x_1^5 - x_1^4 x_2 \\ \dot{x}_2 &= x_1 - x_2 - \beta x_1^3,\end{aligned}$$

where α and β are constant.

- Discuss for what value of α the stability of the origin does not depend on β. [1p]
  Solution: When $\alpha \neq 0$, by the principle of stability in the first approximation.
- For the remaining case analyze the stability in terms of β. [2p]
  Solution: When $\alpha = 0$, using the center manifold theory we have that the system is asymptotically stable if $\beta > 0$, unstable if $\beta < 0$, and critically stable if $\beta = 0$.

4. Consider in a neighborhood N of the origin

$$\begin{aligned}\dot{x}_1 &= -x_2 - x_1^3 \\ \dot{x}_2 &= x_1 + x_2^2 + u \\ \dot{x}_3 &= x_1 - x_3^3 \\ y &= x_1.\end{aligned}$$

- Convert the system locally into the normal form. [2p]
  Solution: Let $\xi_1 = x_1,\ \xi_2 = x_2,\ z = x_3$. The rest is omitted.
- Can the high gain output control $u = -ky,\ k > 0$ stabilize the system locally? [1p]
  Solution: No.
- Is the system exactly linearizable (without considering the output) around the origin? [2p]
  Solution: Yes if for example we choose $\lambda(x) = x_3$.