SF2842: Geometric Control Theory, 2012
Answers of Homework 2

December 7, 2012

1.

(a) $Wriggle = [Steer, Drive] = \begin{pmatrix} \sin(x_3 + x_4) \\ -\cos(x_3 + x_4) \\ -\cos(x_4) \\ 0 \end{pmatrix}$, and

$$[Steer, Wriggle] = \begin{pmatrix} -\cos(x_3 + x_4) \\ -\sin(x_3 + x_4) \\ -\sin(x_4) \\ 0 \end{pmatrix} = -g_1,$$

$$[Wriggle, Drive] = \begin{pmatrix} -\sin(x_3) \\ \cos(x_3) \\ 0 \\ 0 \end{pmatrix} = Slide.$$

(b) No, since $[g_2, g_1] = Wriggle \notin span\{g_1.g_2\}$.

(c) $R_2(x) = span\{g_1, g_2, Wriggle, Slide\}$, and the determinant of the matrix $\begin{pmatrix} g_2 & g_1 & Wriggle & Slide \end{pmatrix}$ is -1, which means $R_c(x) = R_2(x)$ and $\dim R_c(x) = 4$. So the system is locally strongly accessible. Because the f function is zero in this system, it is also controllable.

2.

(a) We just need the average of the initial positions to be $(0, 0)$, which leads to the condition

$$x_3^{(1)} + x_4^{(1)} = 2, \text{ and } x_3^{(2)} + x_4^{(2)} = 0.$$

(b) The system

$$\dot{x} = \begin{pmatrix} -2k & k & 0 & k \\ k & -2k & k & 0 \\ 0 & k & -2k & k \\ k & 0 & k & -2k \end{pmatrix} x$$

with $k \geq 1.5$ fulfills all the conditions.

3.

(a) When $\alpha < -1$, the system is asymptotically stable. When $\alpha > -1$, the system is unstable.

(b) When $\alpha = -1$, the system can be written as

$$\dot{w} = -w^3 + O(w^4)$$

on the center manifold, which is asymptotically stable.

Therefore the original system is asymptotically stable when $\alpha = -1$.

4.

(a) The relative degree of the system is 2. Let $\xi_1 = y = x_1 + x_2$, $\xi_2 = \dot{\xi}_1 = x_1 + x_2 + x_3$ and $z = 2x_1 - x_3$. The normal form of the system will be

$$\begin{aligned}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \xi_1 + (\frac{3}{2}\xi_1 - \frac{1}{2}\xi_2 + z)^3 + 2u \\
\dot{z} &= -z + 2\xi_1 + 3(\frac{3}{2}\xi_1 - \frac{1}{2}\xi_2 + z)^3
\end{aligned}$$

(b) The zero dynamics is $\dot{z} = -z + 3z^3$, which is a stable system.

Therefore, the system can be locally stabilized around the origin.

(c) $ad_f g = [f, g] = \begin{pmatrix} 1 + 3x_2^2 \\ -3 - 3x_2^2 \\ 2 - 3x_2^2 \end{pmatrix}$ and $ad_f^2 g = [f, ad_f g] = \begin{pmatrix} -6x_2 \\ 6x_2 \\ 6x_2 \end{pmatrix} \notin span\{g, ad_f g\}$, which implies $span\{g, ad_f g\}$ is not involutive, and the system cannot be exactly linearized around the origin.