SF2842: Geometric Control Theory, 2014
Answers of Homework 3

March 23, 2014

1. **Solution**:

2. **Solution**:

a. True. This can be proved by showing $\frac{\partial}{\partial t}\|x_i - x_0\|^2$ is negative when x_i is already on the boundary, namely, $\|x_i - x_0\| = r$

b. False. A counter example can be

$$\dot{x}_1 = -x_1,$$
$$\dot{x}_2 = x_2 + u.$$

3. **Solution**:

a. The first approximation gives:

$$\dot{x}_1 = \alpha x_1,$$
$$\dot{x}_2 = x_1 - 2x_2,$$

and is:

- exponentially stable when $\alpha < 0 \Rightarrow$ The system is exponentially stable;
- unstable when $\alpha > 0 \Rightarrow$ The system is unstable;
- critical stable when $\alpha = 0 \Rightarrow$ The stability of the system depends on β.

b. When $\alpha = 0$the system can be written as:

$$\dot{\omega} = -\beta\omega^7 + O(\beta^2\omega^9)$$

on the center manifold, and is

- exponentially stable when $\beta > 0 \Rightarrow$ The system is exponentially stable;
- unstable when $\beta < 0 \Rightarrow$ The system is unstable;
- stable when $\beta = 0 \Rightarrow$ The system is stable (but not asymptotically).

4. **Solution**:

a. Let $\xi_1 = x_1$, $\xi_2 = x_2 - x_1^3$, and $z = x_3 e^{-x_2}$, then we can write the system in its normal form:

$$\begin{aligned}
\dot{z} &= (\xi_1^3 + \xi_2)z - e^{2(\xi_1^3+\xi_2)} z^3 + \xi_1^3 e^{-(\xi_1^3+\xi_2)} \\
\dot{\xi}_1 &= \xi_2, \\
\dot{\xi}_2 &= z^2 e^{2(\xi_1^3+\xi_2)} - \xi_2 - \xi_1^3 - 3\xi_1^2\xi_2 + u.
\end{aligned}$$

Zero dynamics: $\dot{z} = -z^3$.

b. Yes, since the zero dynamics is stable.

c. No, since the matrix $\begin{pmatrix} g(x0) & ad_f g(x_0) & ad_f^2 g(x_0) \end{pmatrix}$ has only rank 2.