KTH Matematik

SF2842: Geometric Control Theory

Solution to Homework 3

Due March 8, 16:50pm, 2017

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1. Consider the system

$$\dot{x} = g_1 u_1 + g_2 u_2,$$

where

$$g_1 = \begin{pmatrix} \cos(x_3 + x_4) \\ \sin(x_3 + x_4) \\ \sin(x_4) \\ 0 \end{pmatrix} \quad g_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

One can view this as a more complex vehicle steering system. Define:

$$Drive = g_1, \quad Steer = g_2, \quad Wriggle = [Steer, Drive], \quad Slide = \begin{pmatrix} -\sin(x_3) \\ \cos(x_3) \\ 0 \\ 0 \end{pmatrix},$$

where $[\cdot, \cdot]$ is the Lie Bracket.

(a) What is $[Steer, Wriggle]$ and $[Wriggle, Drive]$? [2p]

(b) Is the distribution $span\{g_1, g_2\}$ involutive? [1p]

(c) Show that the system is locally strongly accessible and controllable. [3p]

Solution: omitted

2. Determine and justify if each of the following statements is true or false.

(a) Consider the multi-agent system: $\dot{x}_i = u_i$, $x_i \in R$, $u_i \in R$, $i = 1, \cdots, N$. Define the output of the system as $y = x_1$. If $u_i = \sum_{j \in N_i}(x_j - x_i)$ is applied, the system is observable if the interaction graph is connected. [3p]

Solution: False, take for example, the case of three agents ($\dot{x}_i = u_i$) with complete graph.

(b) Consider a smooth nonlinear control system

$$\dot{x} = f(x) + g(x)u,$$

where $f(0) = 0$. If $x = 0$ is exponentially stabilizable by a Lipschitz continuous feedback control, then the system must be exactly linearizable around the origin. [2p]

Solution: False. one can easily construct a counter example.

3. Consider

$$\begin{aligned}\dot{x}_1 &= \alpha x_1 + x_2 + 2x_1^2 + x_1^3 x_2 \\ \dot{x}_2 &= -x_2 + \beta x_1^2,\end{aligned}$$

where α and β are constant.

(a) Discuss for what value of α the stability of the origin does not depend on β. [1p]

Solution: $\alpha \neq 0$.

(b) For the remaining case analyze the stability in terms of β. [2p]

Solution: asymptotically stable if $\beta = -2$, otherwise unstable.

4. Consider in a neighborhood N of the origin

$$\begin{aligned}\dot{x}_1 &= x_2^3 + e^{x_3} u \\ \dot{x}_2 &= -x_2 + \alpha x_1^3 + x_2^2 \\ \dot{x}_3 &= -x_2^3 + e^{x_3} u \\ y &= x_3,\end{aligned}$$

where α is a constant.

(a) Convert the system into the normal form. .... (3p)
Solution: The system has rel. degree 1. We can let $\xi = x_3$, $z_1 = x_1 - x_3$, $z_2 = x_2$.

(b) Design a feedback control to stabilize the nonlinear system for the case when the zero dynamics is asymptotically stable. .... (2p)
Solution: The zero dynamics is asymp. stable when $\alpha < 0$.

(c) Show the system without the output is not exactly linearizable........... (1p)
Solution: We can show that the linearized system is not controllable.