KTH Matematik

Solution to Final Exam of SF2842 Geometric Control Theory

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Allowed written material: All course material (except the old exams and their solutions) and β mathematics handbook.

Solution methods: All conclusions must be properly motivated. Note: the problems are not necessarily ordered in terms of difficulty.

Note! Your personnummer must be stated on the cover sheet. Number your pages and write your name on each sheet that you turn in!

Preliminary grades: 31 points give grade C, 37 points B and 43 points give grade A.

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1. Determine if each of the following statements is *true* or *false* and **motivate** (no motivation no score) your answer briefly (for example, to show a statement is false, a counter-example is enough).

(a) Consider a square linear system that is *minimal*

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (1)$$

where $x \in R^n$, $u \in R^m$, $y \in R^m$.

If $V^* = 0$, then system (1) does not have any transmission zero. ....(2p)

answer: False, unless the system has relative degree.

(b) If $R^* = 0$, then system (1) has a relative degree $(r_1, \cdots, r_m)$. ....(2p)

answer: False.

(c) Consider an exo-system $\dot{w} = Sw, y_r = qw$, where *dimension* $(w) <$ *dimension* (x), all the eigenvalues of S have non-negative real parts and the pair (q, S) is observable. If we define $e = y - qw$ in system (1) and assume $m = 1$, then the full information output regulation problem is solvable at least for some output matrix C. ....(2p)

answer: True.

(d) Consider a nonlinear single-input single-output system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned}$$

where $x \in R^n$, $f, g, h \in C^\infty$ and $f(0) = 0,\ h(0) = 0$. If the system has relative degree n around the origin, then the system is asymptotically stabilizable by a state feedback. ....(2p)

answer: True, since it is exactly linearizable.

(e) Consider

$$\dot{x} = \sum_{i=1}^{m} g_i(x)u_i,$$

where $x \in R^n$ and $m < n$. If the system is controllable then the distribution $span\{g_1(x), ..., g_m(x)\}$ is not involutive. . . . . (2p)
answer: True, this is a necessary condition.

2. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} u \\ y &= (0\ 1\ 0\ 0)x. \end{aligned}$$

(a) Find V^*. . . . . (4p)
answer: omitted.

(b) Find R^* contained in V^*. . . . . (3p)
answer: omitted.

(c) Can we find a friend F of V^*, such that $A + BF$ has all the eigenvalues with negative real parts? . . . . (3p)
answer: omitted.

3. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 1 & 1 & 3 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \\ 1 & 0 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 & -1 \\ 1 & 1 \\ \alpha_1^2 & 0 \\ 0 & \alpha_2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x. \end{aligned}$$

Design coefficients α_1, α_2 such that the following two conditions are both satisfied

(a) the noninteracting control problem is solvable, ... (5p)
and

(b) the zero dynamics is asymptotically stable. . . . . (5p)
answer: omitted.

4. Consider a control system subject to disturbance:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -x_1 - 2x_2 - x_3 + u + w_1\\ \dot{x}_3 &= \alpha x_3 - 2u\\ y &= x_1,\end{aligned}$$

where w_1 is an unknown nonzero constant (disturbance).

(a) Is the disturbance decoupling problem (DDP) solvable? .... (2p)
answer: No, since the disturbance channel is not in V^*.

(b) When $u = 0$, show that if $\alpha < 0$, then for any given initial condition, the output converges to a constant as $t \to \infty$. .... (3p)
answer: This can be shown by letting $\bar{x}_1 = x_1 - w_1$ and showing the system is asymptotically stable under the new state dynamics.

(c) Show that for almost all values of α the error feedback output regulation problem is solvable if we choose $y_r = 0$ as the reference output (you do not need to design the controller). What is (are) the value(s) of α such that this problem is not solvable? .... (5p)
answer: Since $\dot{w}_1 = 0$, and the zero is $\alpha - 2$, $\alpha \neq 2$. $\alpha \neq 0$ either, otherwise the augmented system is observable. We also need to check for what α the system may not be controllable.

5. Consider in a neighborhood N of the origin

$$\begin{aligned}\dot{x}_1 &= 2x_1x_2 - x_1 + u\\ \dot{x}_2 &= -x_2 + \alpha x_1^2 + 3x_2^2\\ \dot{x}_3 &= x_1 - u\\ y &= x_3,\end{aligned}$$

where α is a constant.

(a) Convert the system into the normal form. .... (4p)
answer: The system has rel. degree 1. Let $\xi = x_3$, $z_1 = x_1 + x_3$, $z_2 = x_2$. The rest is omitted.

(b) Analyze the stability of the zero dynamics in terms of α. .... (4p)
answer: $\alpha = 0$: critically stable; $\alpha < 0$, asymptotically stable. $\alpha > 0$, unstable.

(c) Design a feedback control to stabilize the nonlinear system for the case when the zero dynamics is asymptotically stable. .... (2p)
answer: For example, $u = x_1 + x_3$.