KTH Matematik

Solution to Final Exam of SF2842 Geometric Control Theory

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Allowed written material: All course material (except the old exams and their solutions) and β mathematics handbook.

Solution methods: All conclusions must be properly motivated. Note: the problems are not necessarily ordered in terms of difficulty.

Note! Your personnummer must be stated on the cover sheet. Number your pages and write your name on each sheet that you turn in!

Preliminary grades: 31 points give grade C, 37 points B and 43 points give grade A.

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1. Determine if each of the following statements is *true* or *false* and **motivate** (no motivation no score) your answer briefly (for example, to show a statement is false, a counter-example is enough).

(a) Consider a linear system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (1)$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$.
If system (1) is observable, then $V^* = 0$. ....(2p)
Answer: Obviously false.

(b) Let $p = m$. If system (1) does not have any relative degree $(r_1, \cdots, r_m)$, then it does not have any (transmission) zero. ....(2p)
Answer: Obviously false.

(c) Consider a controllable and observable linear system

$$\begin{aligned} \dot{x} &= Ax + Bu + Ew \\ \dot{w} &= Sw \\ y &= Cx, \end{aligned}$$

where w is disturbance. If ImE is not contained in V^*, then the full information output regulation problem (here $y_r = 0$ is the reference output) is never solvable. (2p)
Answer: False, see problem 4.

(d) Consider

$$\dot{x} = f(x) + g(x)u,$$

where $x \in R^n, u \in R, f, g \in C^\infty$ and $f(0) = 0$. If $x = 0$ can be asymptotically stabilized by a feedback control $u = \alpha(x)$, then the nonlinear system is controllable. . . . . (2p)

Answer: Obviously false, for example $g(x)$ can be zero.

(e) Consider a nonlinear single-input single-output system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned}$$

where $x \in R^n$, $f, g, h \in C^\infty$ and $f(0) = 0,\ h(0) = 0$. If the system does not have relative degree n around the origin, then it is not exactly linearizable around the origin. . . . . (2p)

Answer: False, since there might be other choices of output.

2. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 1 & -1 \end{pmatrix} x + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} u \\ y &= (1\ 0\ -1\ 0)x. \end{aligned}$$

(a) Find V^*. . . . . (4p)

Answer: omitted.

(b) Find R^* contained in V^*. . . . . (3p)

Answer: omitted.

(c) Can we find a friend F of V^*, such that $A + BF$ has all the eigenvalues with negative real parts? . . . . (3p)

Answer: omitted.

3. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & -3 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ \alpha & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x, \end{aligned}$$

where α is a real constant.

(a) For what value of α is the noninteracting control problem solvable? . . . . . (4p)

Answer: omitted.

(b) What is the (transmission) zero(s) of the system when the noninteracting control problem is solvable? .... (3p)

Answer: omitted.

(c) Suppose now the second output y_2 is taken away from the system, namely only y_1 is kept. What is the (transmission) zero(s) of the system now if $\alpha = 1$? (3p)

Answer: omitted.

4. Consider a control system subject to disturbance:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_1 - 2x_2 + x_3 + u + w_1 \\
\dot{x}_3 &= \alpha x_3 + 2u \\
y &= x_1,
\end{aligned}
$$

where w_1 is an unknown nonzero constant (disturbance).

(a) Is the disturbance decoupling problem (DDP) solvable? .... (2p)

Answer: No, since the disturbance channel is not in V^*

(b) When $u = 0$, show that if $\alpha < 0$, then for all initial conditions, $y(t) \to w_1$ as $t \to \infty$. .... (3p)

Answer: This can be shown by letting $\bar{x}_1 = x_1 - w_1$ and showing the system is asymptotically stable under the new state dynamics.

(c) For what value of α is the error feedback output regulation problem guaranteed to be solvable if we choose $y_r = 0$ as the reference output (you do not need to design the controller)? .... (5p)

Answer: Since $\dot{w}_1 = 0$, and the zero is $\alpha - 2$, $\alpha \neq 2$. $\alpha \neq 0$ either, otherwise the augmented system is observable. We also need to check for what α the system may not be controllable

5. Consider in a neighborhood N of the origin

$$
\begin{aligned}
\dot{x}_1 &= -x_3^3 + u \\
\dot{x}_2 &= -x_2 + \alpha x_3^2 + 3x_2^2 - u \\
\dot{x}_3 &= x_2 x_3 + u \\
y &= x_1,
\end{aligned}
$$

where α is a constant.

(a) Convert the system into the normal form. .... (4p)

Answer: The system has rel. degree 1. Let $\xi = x_1$, $z_1 = x_2 + x_1, z_2 = x_3 - x_1$. The rest is omitted.

(b) Analyze the stability of the zero dynamics in terms of α. .... (4p)

Answer: $\alpha < -1$, asymptotically stable. $\alpha \geq -1$, unstable.

(c) Consider the same system but without any output. Is it exactly linearizable around the origin? (Hint: one does not have to compute any Lie bracket) (2p)

Answer: No, since the (approximately) linearized system is not controllable.