KTH Matematik

Exam of SF2842 Geometric Control Theory

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Allowed written material: All course material (except the old exams and their solutions) and β mathematics handbook.

Solution methods: All conclusions must be properly motivated. Note: the problems are not necessarily ordered in terms of difficulty.

Note! Your personnummer must be stated on the cover sheet. Number your pages and write your name on each sheet that you turn in!

Preliminary grades: 31 points give grade C, 37 points B and 43 points give grade A.

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1. Determine if each of the following statements is *true* or *false* and **motivate** (no motivation no score) your answer briefly (for example, to show a statement is false, a counter-example is enough).

(a) Consider a linear time-invariant system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$.
Given an x_0, there exists an F such that $Ce^{(A+BF)t}x_0 = 0 \ \forall t \geq 0$ if and only if $x_0 \in V^*$. .... (2p)

(b) Let $p = m$. If system (1) does not have any relative degree $(r_1, \cdots, r_m)$, then it does not have any (transmission) zero. .... (2p)

(c) Consider a controllable and observable linear system

$$\begin{aligned} \dot{x} &= Ax + Bu + Ew \\ \dot{w} &= Sw \\ y &= Cx, \end{aligned}$$

where w is disturbance. If $Im\ E$ is not contained in V^*, then the full information output regulation problem (here $y_r = 0$ is the reference output) is never solvable. (2p)

(d) Consider

$$\dot{x} = f(x) + g(x)u,$$

where $x \in R^n, u \in R, f, g \in C^\infty$ and $f(0) = 0$. If $x = 0$ can be asymptotically stabilized by a feedback control $u = \alpha(x)$, then the nonlinear system is controllable. .... (2p)

(e) Consider a nonlinear single-input single-output system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned}$$

where $x \in R^n$, $f, g, h \in C^\infty$ and $f(0) = 0,\ h(0) = 0$. If the system does not have relative degree n around the origin, then it is not exactly linearizable around the origin. .... (2p)

2. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 1 & -1 \end{pmatrix} x + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} u \\ y &= (1\ 0\ -1\ 0)x. \end{aligned}$$

(a) Find V^*. .... (5p)

(b) Find R^* contained in V^*. .... (3p)

(c) Given two distinct points $x_1,\ x_2$ in V^*, describe the condition on them such that there exists a control $u(t)$ that drives the system from $x(0) = x_1$ to $x(T) = x_2$ for any given $T > 0$ while the whole trajectory connecting the two points lies in V^*. .... (2p)

3. Consider the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & -3 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ \alpha & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x, \end{aligned}$$

where α is a real constant.

(a) For what value of α is the noninteracting control problem solvable? ..... (4p)

(b) What is the (transmission) zero(s) of the system when the noninteracting control problem is solvable? .... (3p)

(c) Suppose now the second output y_2 is taken away from the system, namely only y_1 is kept. What is the (transmission) zero(s) of the system now if $\alpha = 1$? (3p)

4. Consider a control system subject to disturbance:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -x_1 - 2x_2 + x_3 + u + w_1\\ \dot{x}_3 &= \alpha x_3 + 2u\\ y &= x_1,\end{aligned}$$

where w_1 is an unknown nonzero constant (disturbance).

(a) Is the disturbance decoupling problem (DDP) solvable? .... (2p)

(b) When $u = 0$, show that if $\alpha < 0$, then for all initial conditions, $y(t) \to w_1$ as $t \to \infty$. .... (3p)

(c) For what value of α is the error feedback output regulation problem guaranteed to be solvable if we choose $y_r = 0$ as the reference output, i.e. $e = y$. (you do not need to design the controller)? .... (5p)

5. Consider in a neighborhood N of the origin

$$\begin{aligned}\dot{x}_1 &= -x_3^3 + u\\ \dot{x}_2 &= -x_2 + \alpha x_3^2 + 3x_2^2 - u\\ \dot{x}_3 &= x_2 x_3 + u\\ y &= x_1,\end{aligned}$$

where α is a constant.

(a) Convert the system into the normal form. .... (4p)

(b) Analyze the stability of the zero dynamics in terms of α. .... (4p)

(c) Consider the same system but without any output. Is it exactly linearizable around the origin? .... (2p)