Solution to

Exam of SF2842, March 19 2014

1. a. True, by virtue of the definition for V^*.

   b. False. One can easily construct counter example using the system matrix.

   c. False. Counter example can be found in the Compendium (Example 7.1)

   d. False, for example $\dot{x} = f(x)$ is already asym. stable.

   e. False, since there might exist $\lambda(x)$, s.t. $y = \lambda(x)$ will give relative degree n.

2. a. $V^* = \text{span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

   b. $R^* = V^*$

   c. $x_1, x_2 \in R^*$

3. a. $\alpha \neq 1$

   b. Asymptotically stable when $\alpha \neq 1$.

   c. $s = -1, -2$.

4. a. No, since $E = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \notin V^*$.

b. When $\alpha < 0$, the corresponding A matrix is a stable matrix. Thus $y(t) = C\Pi w_1$ in the steady state. We can easily compute that $C\Pi = 1$ for this system.

c. $\alpha \neq 0$
$\alpha \neq 2$
$\alpha \neq 1$ (for controllability)

5. a. $\xi = x_1$, $z_1 = x_2 + x_1$, $z_2 = x_3 - x_1$ (there are other ways to choose z)

b. $\dot{z}_1 = -z_1 + 3z_1^2 + \alpha z_2^2 - z_2^3$
$\dot{z}_2 = z_1 z_2 + z_2^3$

$h(z) = \alpha z_2^2 - z_2^3$

$\Rightarrow \dot{w} = (\alpha+1)w^3 - w^4 + \theta$

thus, unstable if $\alpha \geq -1$
asym. stable if $\alpha < -1$.

c. It is not exactly linearizable since the linearized system is not controllable.