KTH Matematik

SF2842: Geometric Control Theory

Homework 1

Due February 7, 16:59, 2017

You may discuss the problems in group (maximal **two** students in a group), but each of you **must** write and submit your own report. Write the name of the person you cooperated with.

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1. Consider the system

$$\begin{aligned} \dot{x} &= Ax + Bu = \begin{pmatrix} -2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \end{pmatrix} x + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} u \\ y &= Cx = (1\ 0\ 0\ 0)x. \end{aligned}$$

(a) Is the system observable? ..... (1p)

(b) Compute $\mathcal{V}^*$ and $\mathcal{R}^*$ contained in $\mathcal{V}^*$, and find (parameterize) ALL friends F of $\mathcal{V}^*$. ..... (3p)

(c) Let $A_F = A + BF$, and $\Omega_F = (C^T, A_F^T C^T, \cdots, (A_F^3)^T C^T)^T$, find an F that maximizes the dimension of $ker\ \Omega_F$ and $A + BF$ is a stable matrix. ..... (2p)

2. Consider the system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned}$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$. Determine if each of the following statements is true or false. You must justify your answer!

(a) If the dimension of V^* is greater than 0, then for any friend F of V^*, $(C, A+BF)$ is not observable. ..... (2p)

(b) Suppose the system is controllable, then any controlled invariant subspace is also a controllability subspace. ..... (2p)

3. Consider a SISO

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned}$$

where $g(s) = C(sI - A)^{-1}B = \frac{s^m + p_1 s^{m-1} + \cdots + p_m}{s^n + d_1 s^{n-1} + \cdots + d_n}$

Use Ω^* algorithm to show that the dimension of V^* is m. ..... (3p)

4. Consider

$$\begin{aligned}
\dot{x}_1 &= x_1 + x_3 + u_1 \\
\dot{x}_2 &= x_2 - x_3 + u_1 \\
\dot{x}_3 &= -x_3 + 2x_4 + u_2 \\
\dot{x}_4 &= x_1 + \alpha x_2 + x_4 + u_1 \\
y_1 &= x_1 - x_2 \\
y_2 &= x_4,
\end{aligned}$$

where α is a constant.

(a) What is the relative degree for the system? ..... (1p)

(b) Convert the system into the normal form and compute the zero dynamics.(3p)

(c) When $y(t) = 0\ \forall t \geq 0$, does it always imply $\lim_{t\to\infty} x(t) = 0$? ..... (2p)