KTH Matematik

SF2842: Geometric Control Theory

Homework 3

Due March 10, 16:50pm, 2016

You may use min(5,(your score)/4) as bonus credit on the exam

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1. Consider the system

$$\dot{x} = g_1 u_1 + g_2 u_2,$$

where

$$g_1 = \begin{pmatrix} \cos(x_3 + x_4) \\ \sin(x_3 + x_4) \\ \sin(x_4) \\ 0 \end{pmatrix} \quad g_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

One can view this as a more complex vehicle steering system. Define:

$$Drive = g_1, \quad Steer = g_2, \quad Wriggle = [Steer, Drive], \quad Slide = \begin{pmatrix} -\sin(x_3) \\ \cos(x_3) \\ 0 \\ 0 \end{pmatrix},$$

where $[\cdot,\cdot]$ is the Lie Bracket.

- What is $[Steer, Wriggle]$ and $[Wriggle, Drive]$? [2p]

  Solution: $[Steer, Wriggle] = -Drive$, and $[Wriggle, Drive] = Slide$.

- Is the distribution $span\{g_1, g_2\}$ involutive? [1p]

  Solution: No, since $[g_1, g_2] = -Wriggle \notin \text{span}\{g_1, g_2\}$

- Show that the system is locally strongly accessible and controllable. [3p]

  Solution: Omitted.

2. Determine and justify if each of the following statements is true or false.

- Consider the consensus control problem in R^2 with N agents: $\dot{x}_i = u_i,\ x_i \in R^2,\ u_i \in R^2,\ i = 1, \cdots, N$. If the initial positions of the agents are contained in a disc, then as the consensus control $u_i = \sum_{j \neq i}(x_j - x_i)$ is applied, no agent can ever move outside the disc. [2p]

  Solution: True. Suppose agent i is the first agent about to leave the disc. When it is on the boundary of the disc, according to the consensus protocol, there must be at least one other agent located outside the disc so that the derivative will point to the outside. This contradicts with the assumption that agent i is the first moving out.

- Consider a smooth nonlinear control system

$$\dot{x} = f(x) + g(x)u,$$

where $f(0) = 0$. If $x = 0$ is not exponentially stabilizable by a Lipschitz continuous feedback control, then the system is not exactly linearizable around the origin either. [2p]

Solution: True. If the exact linearization is solvable, we can design feed back control that stabilizing the obtained linear system, and therefore can stabilize the original system.

3. Consider

$$\begin{aligned} \dot{x}_1 &= \alpha x_1 + 2x_1^4 - x_1^3 x_2 \\ \dot{x}_2 &= 2x_1 - x_2 - \beta x_1^2, \end{aligned}$$

where α and β are constant.

- Discuss for what value of α the stability of the origin does not depend on β. [1p]

Solution: When $\alpha \neq 0$.

- For the remaining case analyze the stability in terms of β. [3p]

Solution: The system is asymptotically stable when $\beta < 0$, stable (but not asymptotically stable) when $\beta = 0$, and unstable when $\beta > 0$.

4. Consider in a neighborhood N of the origin

$$\begin{aligned} \dot{x}_1 &= x_3 - x_1^5 \\ \dot{x}_2 &= x_1 - (e^{x_2}\cos(x_3) - 1)^3 + \sin(x_3)u \\ \dot{x}_3 &= \cos(x_3)u \\ y &= x_1. \end{aligned}$$

- Convert the system locally into the normal form. [3p]

Solution: Let $\xi_1 = x_1$, $\xi_2 = x_3 - x_1^5$ and $z = e^{x_2}\cos x_3 - 1$, and the normal form is:

$$\begin{aligned} \dot{z} &= -z^3 - z^4 + \xi_1 + z\xi_1 \\ \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= -5\xi_1^4\xi_2 + \cos(\xi_1^5 + \xi_2)u \\ y &= \xi_1. \end{aligned}$$

- Can the system be stabilized locally around the origin? [1p]

Solution: Yes, since the zero dynamics $\dot{z} = -z^3 + z^4$ is asymptotically stable.

- Is the system exactly linearizable (without considering the output) around the origin? [2p]

Solution: Yes.