Some questions on chapter 1.

The most important question of them all: (Except the meaning of life.) Do you have any questions that you want to discuss?

Question 1: Carathedory's definition for a set A to be μ-measurable is that

$$\mu(S) = \mu(S \setminus A) + \mu(S \cap A)$$

for all sets S. But what does that mean?

Question 2: There are many different definitions relating to measures such as ' *"measurable set"*, *"Borel-regular measure"* and *"Radon measure"*. But can you find any good examples of

1. A non measurable set?[1]
2. A measure that is not Borel?[2]
3. A measure that is not Radon?

Can we draw any conclusions from this?

Question 3: I mentioned that integration on a surface S:

$$\int_S \phi(x)dx \qquad \text{for } \phi \in C_c(R^n)$$

could be formulated as an integral over $\mathbb{R}^n$ for some measure μ_S:

$$\int_{\mathbb{R}^n} \phi(x)d\mu_S(x).$$

Can one use the Hausdorff measure $\mathcal{H}^{n-1}$ to explicitly write the measure μ_S?

Question 4: In the discussion of how to find a minimal surface we remarked that there was a problem with convergence of the surfaces S_j if the surfaces where interpreted in a classical way. Look at theorem 4.4 on page 22 and see if you can understand the importance of that theorem in view of the problems with convergence of classical surfaces.

Question 5: What kind of problems might arise in using measures (instead of surfaces) to find minimal surfaces? Is it absolutely clear that interpreting a surface as a measure will painlessly give us all the theory for minimal surfaces that we want?

---

[1] Such an example can only be constructed for the Lebesgue measure by using the axiom of Choice.

[2] I could not come up with any decent examples myself when I tried last night.