INSTRUCTIONS FOR THE EXAMINATION SEMINAR IN GMT.

1. General instructions.

1.1. **Date & Time:** April the 9th from 8-12.

1.2. **Place:** Room 3733 (The usual seminar room.)

1.3. **Format.** The seminar will be four hours long and follow a similar structure as a normal *discussion seminar* with three exceptions:

- The discussion will be in two groups instead of one.
- The conclusion of the discussion will be presented on the black-board.
- Your performance will be assessed - and everyone will present something.

1.4. **Cheating is (mostly) allowed.** [1]

This will be an open book examination. You are allowed to bring any book, article or notes that you think will be of help. If you have Leon Simon's telephone number you may even bring that and a cell phone.

All other imaginable form of, what is usually classified as, cheating is also allowed - with one exception. You are not allowed to cheat (or in any way stretch the truth) on your study log (see below). Anyone that hands in a dishonest study log will be punished with the uttermost severity. If crucifixion is not a sanctioned form of punishment according to KTH's disciplinary handbook any cheating on the study log will be rewarded with a failure of the course. In case crucifixion is a sanctioned form of punishment at KTH the proceedings will take place during the easter weekend with all the usual crucifixion ceremony (crowning with thorns, stabbing with spears etc. see the gospel of Mark 15:16-41). Wine vinegar on a spunge will be offered as refreshment.

2. What you need to do before the seminar.

You will have to do three things in preparation for the examinations seminar.

2.1. **Reading & study log:** You will have to study pages 1-129 in Leon Simon's book. You are also required to fill in this study log.

[1] It is malicious slander to think that cheating is allowed only to make it easier for the examiner to get a coffee refill and a smoke break during the exam...

	Date	Start time	end time	study time
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
Total hours:	XXX	XXX	XXX	_____(hours/minutes)

Your target time is at least 15 hours of study before the exam, I would prefer more than 20 hours but that depends on how much you studied during the term. If you have logged to few study hours you may be asked to take the exam at a later date when you have had a better chance to prepare.

2.2. **Consider the following questions.** Some of the questions during the examination seminar will be based on the following study questions. These questions are also meant to help you to focus on certain parts of the material.

It is very much to ask that you should master the entire reading you are therefore allowed to *opt out* on certain questions. That is, you may choose three questions that you would rather not want to answer. If one of those questions are posed during the seminar you are free not to answer that question. However, if a second *"opted out"* question is asked you will be required to answer that one. Question 16 can not be opted out!

Opt out questions (at most three): _____ and ____.

Question 1: How would you describe the proof of the Riesz representation theorem (Theorem 4.1 on p. 18) in 5-7 minutes.

Question 2: The following question relates to Rademacher's Theorem (Theorem 5.2 on page 30)

(1) Explain how the following classical results are used in the proof of Rademacher's Theorem:
   - Absolutely continuous functions on the line are differentiable a.e.
   - Fubini's Theorem.

(2) Assuming that you have proved (3) on page 31 for a continuous, but not necessarily Lipschitz, function f. Does it follow that grad(f) exists a.e.?

Question 3:

(1) How does the gradient ∇^M on a manifold, defined as on the third line on p. 42, relate to the normal gradient in $\mathbb{R}^{n+k}$?
(2) What is the geometric meaning of the second fundamental form B_y as defined by (7.3) on page 43.

Question 4: How would you go about proving the area formula (Formula (8.4) p. 46). Formulate your answer as you would explain it to a beginning masters student in 5-7 minutes. Try in particular to explain what it means and what makes it natural.

Question 5: What does the *first variation formula* say and what is varied? How does the first variation formula look when $M = \{(x, f(x));\ x \in [0,1]\}$ is the graph of a $C^2([0,1])$ function $f(x)$ and $\frac{\partial \phi(x,y,t)}{\partial t}\Big|_{t=0} = (0, \psi(x))$ for $\psi \in C^1_c([0,1])$ for x, y in a neighborhood of the graph of $f(x)$.

Question 6: Try to draw a picture to explain the idea behind the proof of Theorem 11.8 (the Theorem is stated on p. 62 and the proof on the following pages).

Question 7: Why is the area formula generalizable to countably $n-$rectifiable sets M as in §12 (pp. 66-70). What is it that makes this generalization intuitively acceptable.

Question 8: What is the role of Riesz representation theorem in the proof of Theorem 14.3 on page 72?

WARNING: *Riesz representation theorem is not mentioned in the proof of Theorem 14.3 - however I maintain that it is the most important ingredient in the proof.*

Question 9: Describe how a set of finite perimeter gives rise to a varifold. - Is the opposite, that every varifold gives rise to a set of finite perimeter, true?

Question 10: What is the idea of the proof of the Monotonicity formula. What is monotone in the stationary varifold case when $\underline{\underline{H}} = 0$?

Question 11: How is the monotonicity formula used in Lemma 17.11 (p. 88). What is the geometric idea?

Question 12: What does Lemma 19.5 (p.99) say, how is it related to/important for the proof of Theorem 20.2? (And there went one of your "opt out questions".)

Question 13: Explain the strategy of the proof of Theorem 23.1 (p. 120), what is the idea and geometry of the proof? Once again, imagine that you have 5-7 minutes to explain it to a masters student. You may assume that you have a stationary varifold $\underline{\underline{H}} = 0$.

Question 14: How is the covering theorem used in the proof of Theorem 20.2.

Question 15: Explain how *Harmonic approximation* is used in the proof of Theorem 22.5 on p.114. What is the idea behind the harmonic approximation? How does the 1st variation formula relate to Lemma 21.1? You may assume that the varifold is stationary and that $k = 1$.

Question 16: Try to summarize the entire course by a *"flow diagram of ideas"*.[2] **Can not be opted out!**

Question 17: Look at the questions for seminar 1 and 3 (seminar 2 has gone missing). Seminar questions can be found on the course webpage.

Question 18: Look at the questions for seminar 4 and 5. Seminar questions can be found on the course webpage.

Question 19: Look at the questions for seminar 6 and 7. Seminar questions can be found on the course webpage.

Question 20: Look at the questions for seminar 8 and 9. Seminar questions can be found on the course webpage.

2.3. **Create your own question.** Create one discussion question relating to the text. The question should cover an important aspect of the theory and it should be possible to answer it after around 15 minutes discussion. If you can, make a question whose solution needs a sketch. One or more of your questions may be used during the examination (in which case you get rewarded by knowing the answer beforehand).

You should bring your question to the examination seminar, preferably written in latex.

Do not discuss your question with other course participants.

Good Luck!

[2] I gave you an example of a *"flow diagram of ideas"* in the last seminar.