GMT Seminar 12th February.

Question 1. [Eric Larsson] Let V be a flat unit disk with center $\xi \in \mathbb{R}^3$, and let R be smaller than the radius of the disk. Explain why this is not a counterexample to Lemma 19.1.

Question 2. [Eric Larsson] The first sentence on page 99 states that "It is easy to check that (5) [...] implies that μ_W is invariant under homotheties [...]". How would you check this?

Question 3. [Eric Larsson] The statement of Lemma 19.5 is somewhat difficult to read. Is there a simpler special case which still captures the essence of the lemma?

Question 4. [Andreas Minne and Erik Duse.] On page 95, how does one get the exponent $n/n-1$ in the inequality following "we then get..."?

Question 5. [Andreas Minne and Erik Duse.] This question concerns the last session, namely 17.9(1) Remarks: why is it true that "... hence we can write $V|U = v(M_*, \theta_*)$"?

Question 6. [Andreas Minne and Erik Duse.] How would you explain the reasoning for finding the proofs of the results in Chapter 18 and 19 (for example the proofs of 18.6 Theorem and Lemma 19.5)?

Question 7: Lemma 19.1 on page 95 looks like one of Leon Simon's normal technical mumbo-jumbo Lemmata. But it has a very intuitive geometric interpretation- what? How does this relate to the maximum principle in elliptic PDE?

Question 8: One of the most powerful tools in analysis is the Bolzano-Weierstrass Theorem. For many applications it is enough to find the convergent sub-sequence of a bounded sequence whose existence Bolzano-Weierstrass Theorem assures. However, in Theorem 19.3 on page 97 we assume the full convergence as $\rho \to 0$ (not just convergence for some subsequence $\rho_j \to 0$).

Is this necessary? Can we weaken the assumptions?

Question 9: Again in relation to Theorem 19.3. What does the Theorem say? What is the meaning? How does Theorem 1.3 relate to the theory of minimal surfaces?

Question 10: The "normal" Sobolev inequality in $W_0^{1,1}(U)$ states that there exists a constant C such that

$$\left(\int_U |h(x)|^{\frac{n}{n-1}} dx\right)^{\frac{n-1}{n}} \leq C \int_U |\nabla h(x)| dx.$$

This is usually interpreted as a statement that $W_0^{1,1}(U) \hookrightarrow L^{n/(n-1)}(U)$.

In Theorem 18.6 on page 93 Simon proves the more general Sobolev inequality on a varifold

$$\left(\int_{B_1(0)} |h(x)|^{\frac{n}{n-1}}\, d\mu\right)^{\frac{n-1}{n}} \leq C \int_{B_1} \big(|\nabla^M h(x)| + |H| h(x)\big)\, d\mu.$$

Does this imply that $W_0^{1,1}(V \cap U) \hookrightarrow L^{n/(n-1)}(V \cap U)$ in some meaningful sense?