Questions for GMT seminar 4

December 16, 2014

1 Multiplicity of approximate tangent spaces

1.1 How unique is the multiplicity of approximate tangent spaces?

Suppose the equality in Definition 11.4 (p. 60) holds for all x with some fixed function θ. Is there a simple characterization of all other choices of multiplicities θ for which the equality holds (still globally, for all x)?
Clearly, we can rescale θ by a positive constant. What about rescaling θ with a continuous function? Are there other possible choices?

1.2 Discontinuities of θ

Do discontinuities of the multiplicity function θ have geometric interpretations?

1.3 Quadratic curves (suggested by Simon)

Consider the set

$$Q = \{(x, y) \in \mathbb{R}^2 \mid y = x^2\} \cup \{(x, y) \in \mathbb{R}^2 \mid y = -x^2\}.$$

At what points does it have an approximate tangent space? Which multiplicity functions are possible?

1.4 The topologist's sine curve

Consider the topologist's sine curve

$$S = \{(x, y) \in \mathbb{R}^2 \mid x = 0,\ -1 \leq y \leq 1\} \cup \{(x, y) \in \mathbb{R}^2 \mid 0 < x < 1,\ y = \sin(1/x)\}.$$

At what points does it have an approximate tangent space? Which multiplicity functions are possible?

Question 1: What is the geometric meaning of the "approximate tangent space" (Definition 11.4). How does that relate to the tangent space as defined in the usual way?

Question 2: At the end of page 63 Simon define the distance between two k-dimensional subspaces. What is the meaning of that definition?

Question 3: The most complicated proof of this weeks reading is the proof of Theorem 11.8. What is the idea behind that proof?

Questions from Andreas:

Question 4: Can you construct a counterexample to that claim that the function $y \mapsto H^{N-n}(f^{-1}(y) \cap A)$ is upper semicontinuous, where A is compact, f Lipschitz from $\mathbb{R}^N \to R^n$, and $N > n$.

Question 5: How would you prove that $H^n(O(A)) = H^n(A)$ for any subset A in $\mathbb{R}^n$ (not necessarily measurable) in $\mathbb{R}^n$ and O is a linear orthogonal map.

Question 6: If $f : \mathbb{R}^n \to \mathbb{R}^n$ is Lipschitz, then for any subset A in $\mathbb{R}^n$ and any $s > 0$, $H^s(f(A)) \leq Lip(f)^s H^s(A)$, where $Lip(f)$ is the Lipschitz constant of f.