Distributed Attitude Control of Multi-Agent Formations

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Background

The inspiration for this work comes from the problem of dual arm manipulation, where a robot with two arms is tasked to grasp, lift, carry, and/or manipulate an object, see figure 1. We take a multi-agent systems theoretic approach. Each manipulator is represented by an agent. The number of agents is arbitrary, but their kinematics are less complex than those of manipulators. We disregard the manipulator joint space kinematics and let the states of an agent represent the position of a manipulator's end-effector.

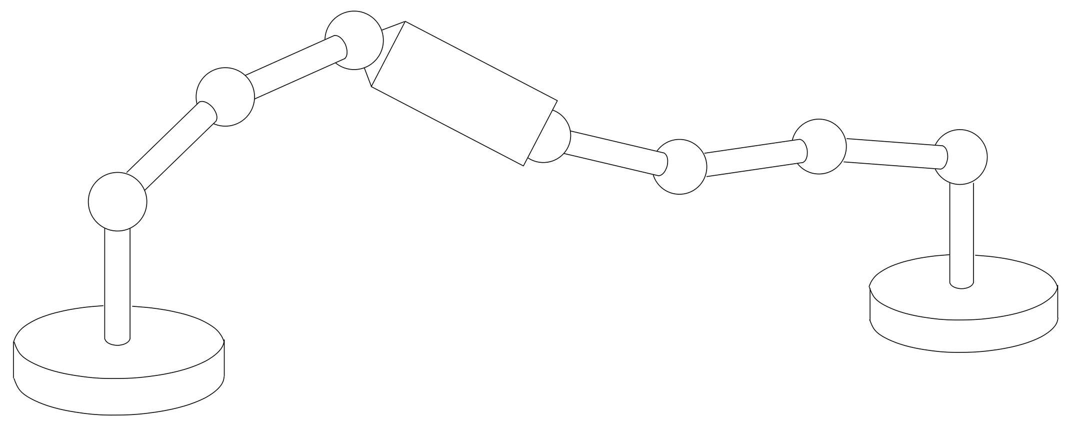


Figure 1. Dual arm manipulation.

Problem Statement

We assume control is carried out at a kinematic level, and specify the dynamics to consist of a single integrator,

$$\dot{\mathbf{x}}_i = \mathbf{u}_i, \quad i = 1, 2, \ldots, n \tag{1}$$

where $\mathbf{x}_i$ and $\mathbf{u}_i$ denotes the position and velocity of the ith agent. Moreover, we require that the constraints

$$\|\mathbf{x}_i - \mathbf{x}_j\| = c_{i,j}, \quad \forall i, j = 1, \ldots, n,$$

where $c_{i,j} \in \mathbb{R}^+$ are constants, are stable. Define the normal

$$\boldsymbol{\psi} = (\mathbf{x}_1 - \mathbf{x}_2) \times (\mathbf{x}_2 - \mathbf{x}_3). \tag{2}$$

The aim is to reach a reference attitude, represented by a unit vector $\mathbf{n} \in \mathbb{R}^3$,

$$\lim_{t\to\infty} \frac{\boldsymbol{\psi}}{\|\boldsymbol{\psi}\|} = \mathbf{n}. \tag{3}$$

Geometric Point of View

The reference attitude consists of two of the three DOF composing the attitude of a rigid body. Let *e.g.* the formation consist of a convex polytope with $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ on one face. The reference attitude is reached when that face belongs to a plane whose normal is $\mathbf{n}$.

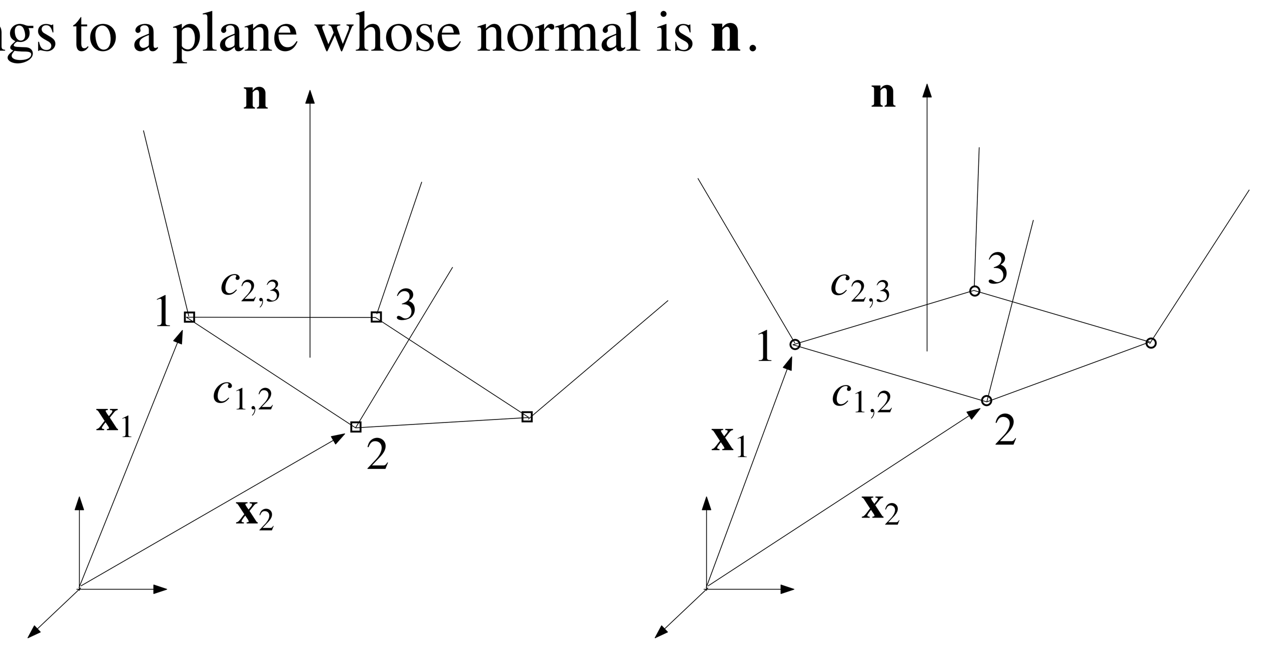


Figure 2. Configuration of agents at initial time (left) and final time (right).

Main Results

Consider the case of three non-collinear agents, 1, 2, and 3. Assume the agents are capable of sensing the relative positions $\mathbf{x}_1 - \mathbf{x}_2$ and $\mathbf{x}_2 - \mathbf{x}_3$. The agents may reach the reference attitude by rotating about the intersection of the reference plane and the plane defined by the agents. The configuration of agents will then rotate about the centroid of $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$.

Main Results, Cont'd.

We propose the control law

$$\mathbf{u}_i = \mathbf{v} + (\mathbf{x}_j + \mathbf{x}_k - 2\mathbf{x}_i) \times \boldsymbol{\omega}, \tag{4}$$

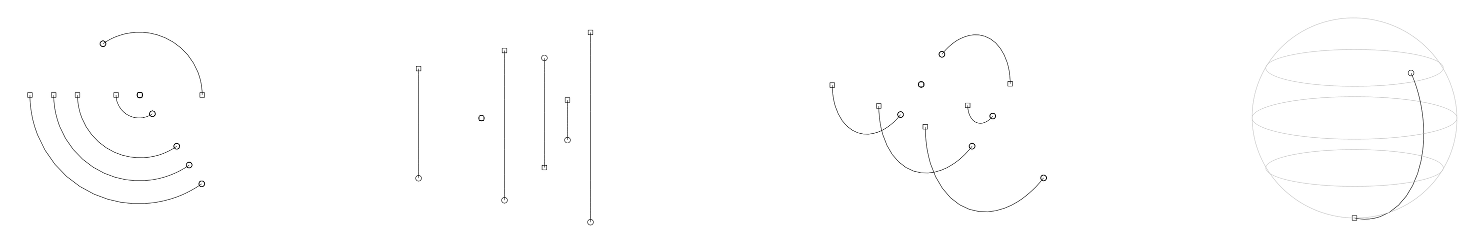
$$\boldsymbol{\omega} = \alpha\, \boldsymbol{\psi} \times \mathbf{n}, \tag{5}$$

where $\alpha \in \mathbb{R}^+$ and $\boldsymbol{\psi}$ is defined by (2). We prove the convergence of (3) using Lyapunov theory. The region of attraction is the whole state space, S^2, except a single bad initial point $\boldsymbol{\psi}(0)/\|\boldsymbol{\psi}\| = -\mathbf{n}$.

The general case with an arbitrary number of agents is addressed using the theory of rigid graphs.

Simulations

The results from a simulation in MATLAB are displayed in figure 1. The differential equations (1) for a system of six coplanar agents with $\mathbf{u}_i$ given by (4)–(5) are solved for $\mathbf{x}_i$ as functions of time using the built-in function `ode45`.



(a) (b) (c) (d)

Figure 3. Trajectories of six agents from different viewpoints, (a)–(c). Trajectory of the normal on the 2-sphere, (d). Squares (□) are initial positions, circles (○) are final positions.

Further Information

For more details of our work, check out

- Wang, L., Markdahl, J., and Hu, X. (2011). Distributed Attitude Control of Multi-Agent Formations. *18th IFAC World Congress*, to appear.

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Main References

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- Whiteley, W. (1997). Rigidity and scene analysis. In J.E. Goodman and J. O'Rourke (ed.), *Handbook of Discrete and Computational Geometry*, 893–916. Chapman and Hall, Boca Raton.

Acknowledgement

This work is partly supported by the Swedish Foundation for Strategic Research (SSF) and the Swedish Research Council (VR).

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