Distributed Attitude Control of Multi-Agent Formations

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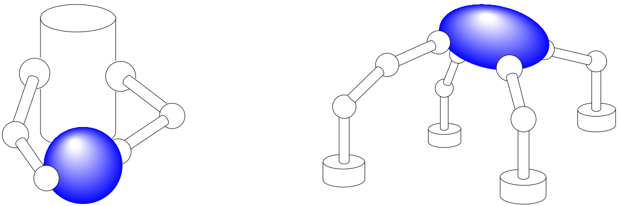
Cooperative Manipulation

Cooperative manipulation

- Perform a manipulation task using multiple manipulators.
- Carry heavier loads, use several tools simultaneously.
- Centralized or decentralized control.



Multi-Agent Model

Every manipulator corresponds to an agent. The agent state is given by the end-effector position.

Manipulator model

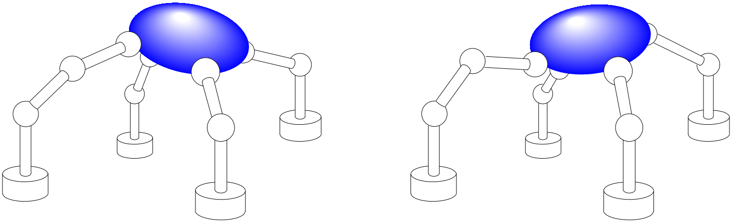
- Manipulator kinematics, $\dot{\mathbf{p}}_i = J_i(\mathbf{q}_i)\dot{\mathbf{q}}_i$, $\dot{\mathbf{q}}_i = \mathbf{u}_i$
- Bilateral constraints at the grasp points

Multi-agent model

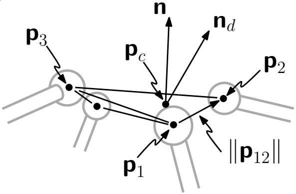
- Single integrator kinematics, $\dot{\mathbf{p}}_i = \mathbf{u}_i$
- Formation of agents should be maintained, *i.e.* distances between agents should be constant

Geometry of the Model

Aim: rotate the object to a desired orientation.



Let



$$\mathbf{p}_{ij} = \mathbf{p}_i - \mathbf{p}_j,$$

$$\mathbf{p}_c = \frac{1}{3}\sum_{i=1}^{3}\mathbf{p}_i,$$

$$\mathbf{n} = \mathbf{p}_{12} \times \mathbf{p}_{23}.$$

Parametrization of Orientation

Commonly used parametrizations of orienation:

- Rotation matrices, $SO(3)$
- Unit quaternions
- Euler angles

We use three vectors: $\mathbf{p}_{12}, \mathbf{p}_{23}$, and $\mathbf{n} = \mathbf{p}_{12} \times \mathbf{p}_{23}$.

- Gram-Schmidt mapping onto $SO(3)$

$$(\mathbf{p}_{12}, \mathbf{p}_{23}, \mathbf{n}) \to \left[\frac{\mathbf{p}_{12}}{\|\mathbf{p}_{12}\|} \quad \frac{\|\mathbf{p}_{12}\|\mathbf{p}_{23}}{\|\mathbf{n}\|} - \frac{(\mathbf{p}_{12} \cdot \mathbf{p}_{23})\mathbf{p}_{12}}{\|\mathbf{p}_{12}\|\|\mathbf{n}\|} \quad \frac{\mathbf{n}}{\|\mathbf{n}\|} \right].$$

- Formalize goal as $\lim_{t \to \infty} \mathbf{n} = \mathbf{n}_d$. This leaves one degree of rotational freedom.

Theory of rigid graphs

Total number of constraints $n(n-1)/2$, requires a complete communications graph?

Theory of rigid graphs

- If there is no set of three collinear points, then $3n-6$ constraints of the type $\|\mathbf{p}_{ij}\| = c_{ij}$ ensure rigidity.
- Rigidity is preserved if $\dot{\mathbf{p}}_{ij}$ belongs to the nullspace of a $|E| \times 3n$ matrix $\mathrm{R}(G)$. E is the edge set in a constraint graph G.

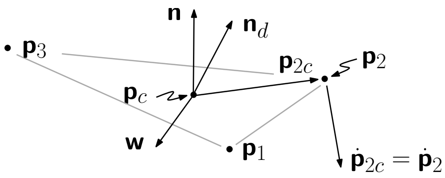
Control Law

The control law

$$\mathbf{u}_i = \mathbf{v} + \mathbf{w} \times \mathbf{p}_{ic},$$
$$\mathbf{w} = \alpha\, \mathbf{n} \times \mathbf{n}_d,$$

gives

$$\dot{\mathbf{p}}_{ij} = \mathbf{w} \times \mathbf{p}_{ij},$$
$$\dot{\mathbf{n}} = \mathbf{w} \times \mathbf{n},$$

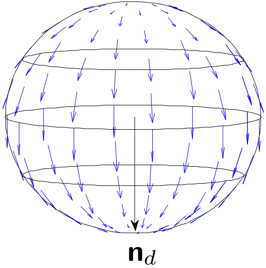


and requires agent i to know $\mathbf{p}_{ic}$, $\mathbf{n}$, and $\mathbf{n}_d$. For example, all agents know $\mathbf{n}_d$ (global information) and are neighbors of the special agents 1, 2, and 3.

Stability Analysis

We prove . . .

- the equilibrium $\mathbf{n} = \mathbf{n}_d$ is almost global asymptotically stable, $\mathbf{n} = -\mathbf{n}_d$ is unstable,



- the convergence rate is locally exponential.

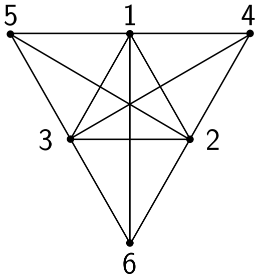
The proof is by Lyapunov theory methods.

An example

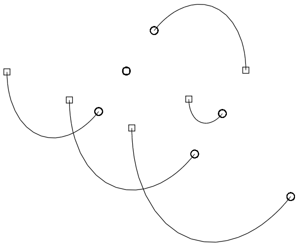
Set up

- Six agents forming an equilateral triangle.
- All agents can sense their relative position with respect to the three special agents 1, 2, and 3.

Communications graph



Simulation



Summary

A distributed control law for rotating a multi-agent formation to any desired orientation.

Future work

- Include manipulator kinematics, $\dot{\mathbf{p}}_i = \mathrm{J}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i$, with rank deficient Jacobian matrix.
- The 2D case.
- Control all three degrees of rotational freedom.

Questions?