Repetition

Fouriertransformen

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$$

- $D(\mathcal{F}) = L^1(\mathbb{R})$
- $\hat{f} \in C_0(\mathbb{R})$, d.v.s. $\hat{f}$ är kontinuerlig och går mot noll när $\omega \to \pm\infty$.

Inversion: $$f(x) = \lim_{A\to\infty} \frac{1}{2\pi} \int_{-A}^{A} \hat{f}(\omega) e^{i\omega x}\, d\omega$$

Om f är

- styckvis kontinuerlig
- $f(x) = \frac{1}{2}\left(f(x^+) + f(x^-)\right) \quad \forall x$
- f har generaliserade höger- och vänsterderivator v.gov.

Om dessutom $\hat{f} \in L^1(\mathbb{R})$ så

är $$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega x} \, d\omega$$

<u>Plancherels formel</u>

$$\|f\|^2_{L^2(\mathbb{R})} = \int_{-\infty}^{\infty} |f(x)|^2 \, dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\hat{f}(\omega)|^2 \, d\omega = \frac{1}{2\pi} \|\hat{f}(\omega)\|^2$$

Jämför med Parsevals formel

$$\sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |f(x)|^2 \, dx$$

$$= \frac{1}{2\pi} \|f\|^2_{L^2}$$

där $$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} \, dx$$

Tillämpningar av Fouriertransformen

- Integralekvationer (faltningar)
- PDE:er i obegränsade områden

3.46 Bestäm a_n och b_n, $n \in \mathbb{N}$, om

$a_0 = 0$, $b_0 = 1$ och

$$\begin{cases} a_{n+1} + b_n = -2n \\ a_n + b_{n+1} = 1, \quad n \in \mathbb{N} \end{cases}$$

L: Vi använder Z-transformen.

BETA s. 328

Z10, $a^n \xrightarrow{Z} \frac{z}{z-a}$

Z12, $na^n \longrightarrow \frac{az}{(z-a)^2}$

Z4, $a_{n+k} \rightarrow z^k A(z) - z^k a_0 - z^{k-1} a_1 - \dots - z a_{k-1}$

där $Z(a_n)(z) = A(z)$, ger

$$\begin{cases} zA(z) - z\underbrace{a_0}_{=0} + B(z) = -2\frac{z}{(z-1)^2} & (1) \\ A(z) + zB(z) - z\underbrace{b_0}_{=1} = \frac{z}{z-1} & (2) \end{cases}$$

V.g.v.

$z\cdot(2)-(1)$ ger

$$\underbrace{z^2B(z)-z^2-B(z)}_{=B(z)(z^2-1)-z^2}=\frac{\overbrace{z^2(z-1)+2z}^{=z(z^2-z+2)}}{(z-1)^2}$$

$$\Leftrightarrow\quad B(z)=\frac{z\cdot(z^2-z+2)}{(z-1)^2(z^2-1)}+\frac{z^2}{z^2-1}$$

$$=\frac{z(z^2-z+2)+\overbrace{z^2(z-1)^2}^{=z\cdot(z^3-2z^2+z)}}{(z-1)^2(z^2-1)}$$

OBS: -1 är en rot till täljaren

$$=\frac{z\cdot(z^3-z^2+2)}{(z-1)^2(z^2-1)}=z\cdot\frac{(z^3-z^2+2)}{(z-1)^3(z+1)}$$

$$=\left\{\begin{array}{l} \quad z^2-2z+2 \\ \hline z^3-z^2+2 \;\; \underline{|\,z+1} \\ -(z^3+z^2) \\ \quad -2z^2+2 \\ -(-2z^2-2z) \\ \hline \quad 0+2z+2 \\ \quad -(2(z+1)) \\ \hline \quad\quad 0 \quad 0 \end{array}\right\}=z\cdot\frac{z^2-2z+2}{(z-1)^3}=$$

v.g.v.

$$= \frac{z^3}{(z-1)^3} - 2 \cdot \frac{z^2}{(z-1)^3} + 2 \cdot \frac{z}{(z-1)^3}$$

BETA s. 328

$$Z14, \quad \frac{z}{(z-a)^{m+1}} \xrightarrow{Z^{-1}} \binom{n}{m} a^{n-m} \theta(n-m)$$

$$Z15, \quad \frac{z^{k+1}}{(z-a)^{m+1}} \xrightarrow{Z^{-1}} \binom{n+k}{m} a^{n+k-m} \theta(n+k-m)$$

$$\Longrightarrow$$

$$b_n = \binom{n+2}{2}\theta(n) - 2\binom{n+1}{2}\theta(n-1) + 2\binom{n}{2}\theta(n-2) =$$

$$= \left\{ n \geq 0, \; \theta(n-k) = \begin{cases} 0, & 0 \leq n \leq k-1 \\ 1, & n \geq k \end{cases} \right\} = \begin{cases} 1, & n = 0 \\ 1, & n = 1 \\ \star, & n \geq 1 \end{cases}$$

$$\star = \frac{(n+2)(n+1)}{2} - 2 \cdot \frac{(n+1)n}{2} + 2 \cdot \frac{n \cdot (n-1)}{2} =$$

$$= \frac{1}{2} \cdot \left(n^2 + 3n + 2 - 2n^2 - 2n + 2n^2 - 2n \right) = \underline{\underline{\frac{1}{2}(n^2 - n + 2)}}$$

OBS: kan inkludera $n=0$ och $n=1$

r.g.v.

Sätter vi in $B(z) = z \cdot \dfrac{z^2-2z+2}{(z-1)^3}$ i (1)
får vi

$$zA(z) = -\frac{2z}{(z-1)^2} - z\,\frac{z^2-2z+2}{(z-1)^3}$$

$$\Leftrightarrow A(z) = \frac{-2(z-1) - z^2+2z-2}{(z-1)^3}$$

$$= -\frac{z^2}{(z-1)^3}$$

z15 i BETA,

$$\frac{z^{k+1}}{(z-1)^{m+1}} \xrightarrow{\mathcal{Z}^{-1}} \binom{n+k}{m} a^{n+k-m}\,\theta(n+k-m)$$

ger, då $\theta(n-1)=1$ när $n \geq 1$, $a_n = \begin{cases} a_0 = 0, & n=0 \\ -\binom{n+1}{2} = -\frac{n^2+n}{2}, & n \geq 1 \end{cases}$

<u>Svar:</u> $\{a_n\}_{n=0}^{\infty} = \left\{-\frac{n^2+n}{2}\right\}_{n=0}^{\infty}$

$\{b_n\}_{n=0}^{\infty} = \left\{\frac{n^2-n+2}{2}\right\}_{n=0}^{\infty}$

— * —

3.53 Bestäm $x(n)$ då $n=0,1,2,\ldots$

så att

$$x(n)+2\sum_{k=0}^{n}(n-k)x(k)=2^n;\quad n=0,1,\ldots$$

L: V.L. ovan innehåller en faltning, d.v.s.

$$x(n)+2(x*f)(n)=2^n$$

där $f(n)=n$

BETA s. 327, 328.

Z8 $\quad x*f \xrightarrow{Z} Z(x)\cdot Z(f)=X\,F$

Z10 $\quad a^n \xrightarrow{Z} \dfrac{z}{z-a}$

Z12 $\quad na^n \xrightarrow{Z} \dfrac{az}{(z-a)^2}$

ger

v.g.v.

$$X(z) + 2X(z) \cdot \frac{z}{(z-1)^2} = \frac{z}{z-2}$$

$$\Leftrightarrow X(z)\left(1 + \frac{2z}{(z-1)^2}\right) = X(z)\left(\frac{\overbrace{(z-1)^2+2z}^{=z^2+1}}{(z-1)^2}\right) = \frac{z}{z-2}$$

$$\Leftrightarrow X(z) = \frac{z(z-1)^2}{(z^2+1)(z-2)}$$

$$= z \cdot \frac{z^2-2z+1}{(z^2+1)(z-2)} = \left\{ \begin{array}{l} \text{Partialbråks-} \\ \text{uppdelning} \end{array} \right\}$$

$$= \frac{z^2-2z+1}{(z^2+1)(z-2)} = \frac{Az+B}{z^2+1} + \frac{C}{z-2}$$

$$\Rightarrow z^2-2z+1 = (Az+B)(z-2) + C(z^2+1)$$

$$= (A+C)z^2 + (B-2A)z + (C-2B)$$

$$\begin{cases} A+C=1 \Leftrightarrow C=1-A \rightarrow A=-2B \\ B-2A=-2 \Leftrightarrow A=\frac{B}{2}+1 \quad -2B=\frac{B}{2}+1 \Leftrightarrow B=-\frac{2}{5} \\ C-2B=1 \Leftrightarrow C=1+2B \end{cases}$$

$$\Rightarrow A=\frac{4}{5}$$

$$C=\frac{1}{5}$$

V.g.v.

$$= \frac{4}{5}\frac{z^2}{z^2+1} - \frac{2}{5}\frac{z}{z^2+1} + \frac{1}{5}\frac{z}{z-2}\Big]$$

BETA s. 328

Z10 $\quad \frac{z}{z-a} \xrightarrow{Z^{-1}} a^n$

Z20 $\quad \frac{z}{z^2+a^2} \xrightarrow{Z^{-1}} a^{n-1}\sin\left(\frac{n\pi}{2}\right)$

Z21 $\quad \frac{z^{k+1}}{z^2+a^2} \xrightarrow{Z^{-1}} a^{n+k-1}\sin\left(\frac{(n+k)\pi}{2}\right)\theta(n+k-1)$

ger

$$x(n) = \frac{4}{5}\sin\left(\frac{(n+1)\pi}{2}\right) - \frac{2}{5}\sin\left(\frac{n\pi}{2}\right) + \frac{2^n}{5}$$

$$= \frac{4}{5}\cos\left(\frac{n\pi}{2}\right) - \frac{2}{5}\sin\left(\frac{n\pi}{2}\right) + \frac{2^n}{5}$$

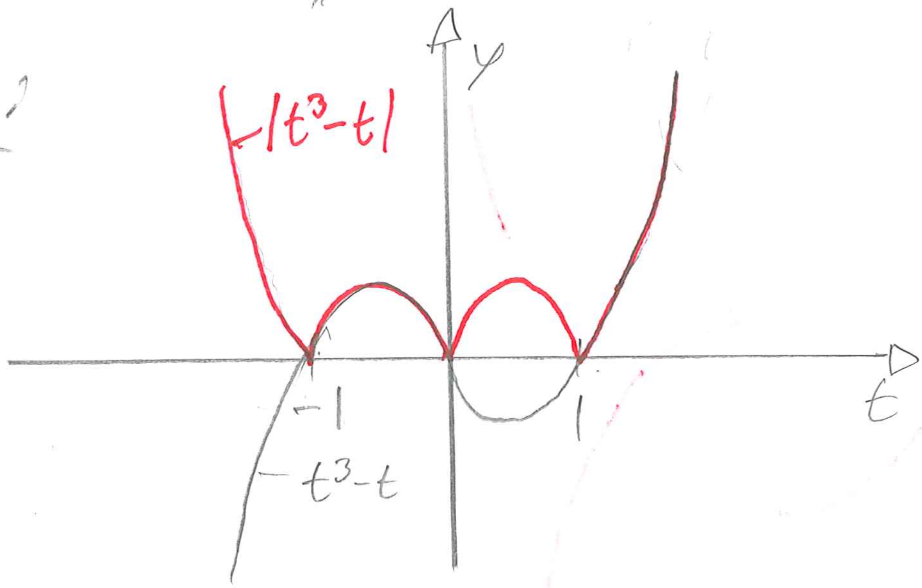
Svar: $\left\{x(n)\right\}_{n=1}^{\infty} = \left\{\frac{4}{5}\cos\left(\frac{n\pi}{2}\right) - \frac{2}{5}\sin\left(\frac{n\pi}{2}\right) + \frac{2^n}{5}\right\}$

2.28 Hitta distributionsderivatorna

f' och f'' om

$$f(t) = |t^3 - t|$$

L:



$$f(t) = (t-t^3)(1-\theta(t+1)) + (t^3-t)(\theta(t+1)-\theta(t))$$

$$+ (t-t^3)(\theta(t)-\theta(t-1)) + (t^3-t)\,\theta(t-1)$$

$$= (t^3-t)(-1+2\theta(t+1)-2\theta(t)+2\theta(t-1))$$

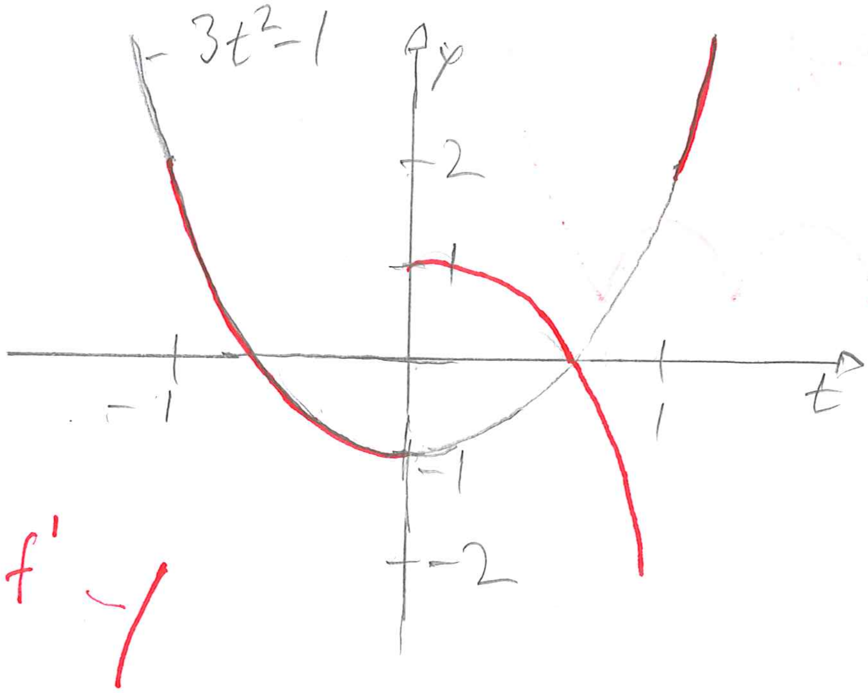
$$\Rightarrow f'(t) = (3t^2-1)(-1+2\theta(t+1)-2\theta(t)+2\theta(t-1))$$

$$+ (t^3-t)(2\delta(t+1)-2\delta(t)+2\delta(t-1)) =$$

v.g.v.

$$= \left\{ \begin{array}{c} f(t)\,\delta(t-a) = f(a)\,\delta(t-a), \\ (t^3-t)\,\delta(t+1)=0,\ (t^3-t)\,\delta(t)=0,\ (t^3-t)\,\delta(t-1)=0 \end{array} \right\} =$$

$$= (3t^2-1)\left(-1+2\theta(t+1)-2\theta(t)+2\theta(t-1)\right)$$

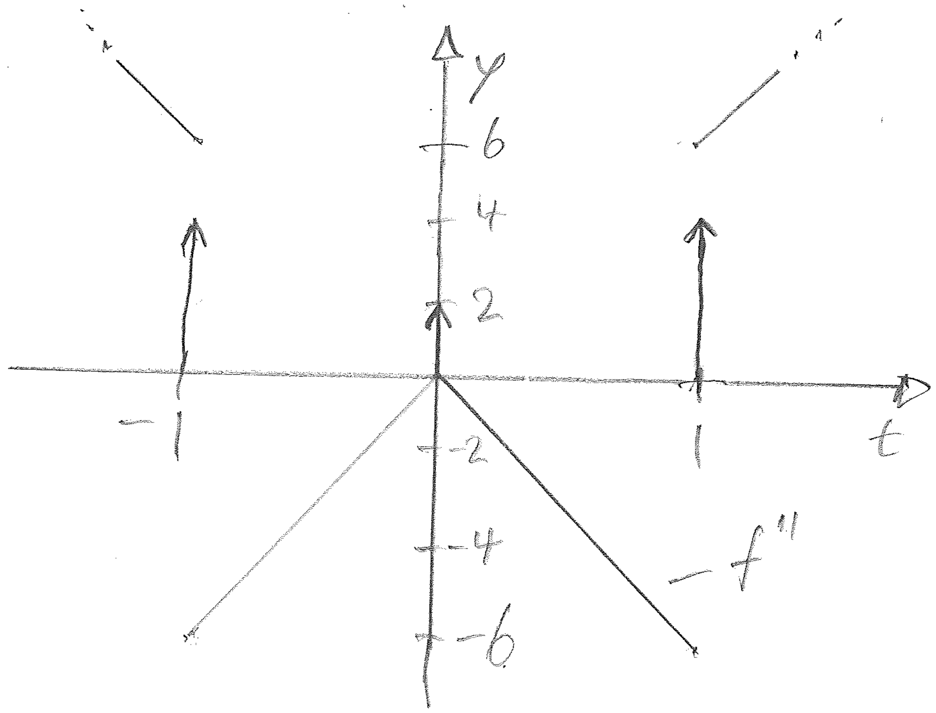


$$f''(t) = 6t\left(-1+2\theta(t+1)-2\theta(t)+2\theta(t-1)\right)$$
$$+ (3t^2-1)\left(2\delta(t+1)-2\delta(t)+2\delta(t-1)\right) =$$

$$= \left\{ \begin{array}{c} (3t^2-1)\,2\delta(t+1) = 4\delta(t+1) \\ (3t^2-1)\,2\delta(t) = -2\delta(t) \\ (3t^2-1)\,2\delta(t-1) = 4\delta(t-1) \end{array} \right\} = \text{v.g.v.}$$

$= 6t\left(-1 + 2\theta(t+1) - 2\theta(t) + 2\theta(t-1)\right)$

$+ 4\delta(t+1) + 2\delta(t) + 4\delta(t-1)$



<u>Svar:</u> $f' = (3t^2-1)(-1+2\theta(t+1) - 2\theta(t) + 2\theta(t-1))$

$f'' = 6t(-1+2\theta(t+1) - 2\theta(t) + 2\theta(t-1))$

$+ 4\delta(t+1) + 2\delta(t) + 4\delta(t-1)$

*

2005-08-30.5

Antag att f är kontinuerlig och tillhör $L^1(\mathbb{R})$. Antag också att

$$\int_{-\infty}^{\infty} f(y) e^{-y^2} e^{2xy} \, dy = 0 \qquad (1)$$

för alla reella x. Visa att $f(x) = 0$ för alla reella x.

L1

$$0 = \int_{-\infty}^{\infty} f(y) e^{-y^2} e^{2xy} \, dy =$$

$$= \left\{ e^{-y^2} e^{2xy} = e^{-y^2+2xy} = e^{-(x-y)^2+x^2} = e^{x^2} e^{-(x-y)^2} \right\} =$$

$$= e^{x^2} \int_{-\infty}^{\infty} f(y) e^{-(x-y)^2} \, dy = \quad \text{v.g.v.}$$

$$= e^{-x^2}\left(f * e^{-x^2}\right) = 0$$

$$\Leftrightarrow \quad f * e^{-x^2} = 0$$

Fouriertransformera m.h.a. BETA

s. 317; 318

F13 $\quad f * g \xrightarrow{\mathcal{F}} \hat{f} \cdot \hat{g}$

F36 $\quad e^{-ax^2} \xrightarrow{\mathcal{F}} \sqrt{\frac{\pi}{a}}\, e^{-\frac{w^2}{4a}}$

$$\Rightarrow \quad \hat{f}(w)\sqrt{\pi}\, e^{-\frac{w^2}{4}} = 0$$

$$\Leftrightarrow \quad \hat{f}(w) = 0 \quad \forall w$$

$$\hat{f} \in L^1(\mathbb{R}) \Leftrightarrow \quad f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{\hat{f}(w)}_{=0} e^{iwx}\, dw = 0$$

$$\therefore \quad f(x) = 0 \quad \forall x \qquad \text{V.S.V}$$

———— * ————

2005-01-12.7

Lös Laplaces ekvation i området $\{(x,y):\ 0\le x\le\pi,\ 0\le y\le\pi\}$ med randvillkoren

$$\begin{cases} u(x,0)=u(0,y)=0 \\ u(x,\pi)=\sin 3x \\ u(\pi,y)=\sin 3y \end{cases}$$

L: Vi delar upp problemet i två delproblem,

$$P_1\begin{cases} \Delta u=0,\quad 0<x,y<\pi & (1) \\ u(x,\pi)=\sin(3x) & (\text{IRV}) \\ u(x,0)=u(0,y)=u(\pi,y)=0 & (\text{HRV}) \end{cases}$$

$$P_2\begin{cases} \Delta u=0,\quad 0<x,y<\pi \\ u(\pi,y)=\sin(3y) \\ u(x,0)=u(0,y)=u(x,\pi)=0 \end{cases}$$

v.g.v.

Om u_1 löser P_1 och u_2 löser P_2

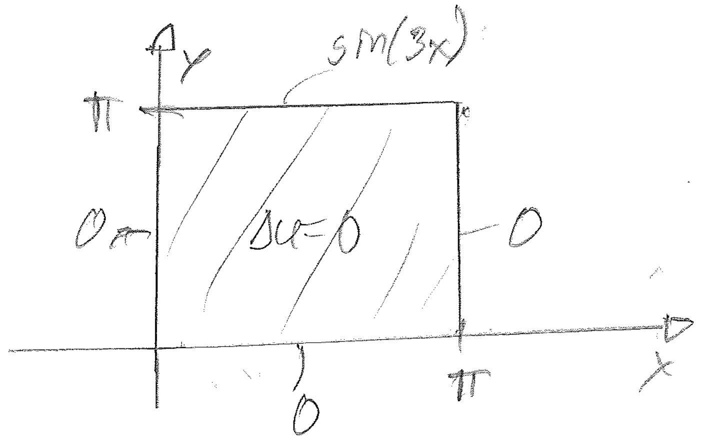
så löser $u = u_1 + u_2$ ursprungsproblemet.

Ansätt u_1 som $u_1(x,y) = X(x)\,Y(y)$.

Insatt i (1) ger det

$$X''(x)\,Y(y) + X(x)\,Y''(y) = 0$$

$$\Rightarrow \quad \frac{X''(x)}{X(x)} = -\frac{Y''(y)}{Y(y)} = -\lambda$$



(HRV) ger

$$\begin{cases} X'' + \lambda X = 0 \\ X(0) = X(\pi) = 0 \end{cases} \qquad \begin{cases} Y'' - \lambda Y = 0 \\ Y(0) = 0 \end{cases}$$

V.g.v.

$$\Rightarrow \quad X(x) = X_n(x) = C_n \sin(nx), \qquad \lambda = \lambda_n = n^2, \; n \in \mathbb{Z}$$

$$Y(y) = Y_n(y) = A_n \cosh(ny) + B_n \sinh(ny)$$

$$Y(0) = A_n = 0 \quad \Rightarrow \quad Y_n(y) = B_n \sinh(ny)$$

Detta ger $\quad u_n(x,y) = X_n(x) Y_n(y) = b_n \sinh(ny) \sin(nx)$

Observera att u_n och u_{-n} är linjärt beroende och att $u_0 = 0$.

$$u(x,y) = \sum_{n=1}^{\infty} u_n(x,y) = \sum_{n=1}^{\infty} b_n \sinh(ny) \sin(nx)$$

löser (1) och (HRV)

$$u(x,\pi) = \sum_{n=1}^{\infty} b_n \sinh(n\pi) \sin(nx) = \{(IRV)\} = \sin(3x)$$

$$\Rightarrow \quad b_n = 0, \quad n \neq 3, \quad b_3 = \frac{1}{\sinh(3\pi)}$$

V.g.v.

Detta ger lösningen

$$u_1(x,y) = \frac{\sinh(3y)\sin(3x)}{\sinh(3\pi)} \quad \text{som} \quad P_1$$

Av symmetriskäl ser vi att

$$u_2(x,y) = \frac{\sinh(3x)\sin(3y)}{\sinh(3\pi)}$$

löser P_2.

Svar: $u = u_1 + u_2$

$$= \frac{\sinh(3y)\sin(3x) + \sinh(3x)\sin(3y)}{\sinh(3\pi)}$$

(Verifiera lösningen)

2008-06-03.3

Beräkna

$$I = \int_0^{\pi} \sin^2(t)\cos^4(t)\, dt$$

m.h.a. Parsevals formel.

L: Låt $f(t) = \sin(t)\cos^2(t)$.

Då kan f skrivas som

$$\sin(t)\cdot(1-\sin^2(t)) = \sin(t) - \sin^3(t)$$

$$= \{\text{BETA s. 128}\} = \sin(t) - \frac{3}{4}\sin(t) + \frac{\sin(3t)}{4}$$

$$= \frac{\sin(t)}{4} + \frac{\sin(3t)}{4}.$$

$\{\sin(kt)\}_{k=1}^{\infty}$ utgör ett

v.g.v.

fullständigt ortogonalt system i

$L^2(0,\pi)$. BETA s. 260 ger

Parsevals formel,

$$\sum_{k=1}^{\infty} N_k c_k^2 = \int_a^b f(t)^2 w(t)\, dt \qquad (1)$$

där $\quad N_k = \|\sin(kt)\|^2 = \int_0^{\pi} \sin^2(kt)\, dt = \frac{\pi}{2}$

$$c_k = \frac{1}{\|\sin(kt)\|^2} \int_0^{\pi} f(t) \sin(kt)\, dt$$

$w(t)=1, \quad a=0, \quad b=\pi$

Vi ser från uttrycket av f att

$c_k=0, \quad k\neq 1,3, \quad c_1 = \frac{1}{4} \quad c_3 = \frac{1}{4}$

Sätter vi in detta (1)

fås $\frac{\pi}{2}\cdot\left(\frac{1}{4}\right)^2 + \frac{\pi}{2}\cdot\left(\frac{1}{4}\right)^2 = \int_0^{\pi} \sin^2(t)\cos^4(t)\,dt$

$$\Leftrightarrow \frac{\pi}{16} = \int_0^{\pi} \sin^2(t)\cos^4(t)\,dt$$

Svar: $I = \frac{\pi}{16}$

2008-06-03.7

Lös
$$\begin{cases} u_{xx} = t u_t, & 0 \le x \le \pi, \quad t \ge 1 \quad (1) \\ u(0,t) = u(\pi,t) = 0, & t \ge 0 \quad (RV) \\ u(x,1) = 4\sin^3(x), & 0 \le x \le \pi \quad (BV) \end{cases}$$

— * —

L: 1) Separation av variabler,

$u(x,t) = X(x)T(t)$ som insatt i (1) ger

$$X''(x)T(t) = t X(x) T'(t).$$

$$\Rightarrow \frac{X''(x)}{X(x)} = \frac{t T'(t)}{T(t)} = -\lambda$$

2-3) (RV) ger problemet

$$\begin{cases} X'' + \lambda X = 0 \\ X(0) = X(\pi) = 0 \end{cases} \quad (2)$$

V.g.v.

(2) har lösningarna

$$X(x) = X_n(x) = B_n \sin(nx) \qquad \lambda = \lambda_n = n^2$$
$$n \in \mathbb{Z}$$

---

4)
$$\frac{t\,T_1'(t)}{T(t)} = -n^2$$

$$\Rightarrow \quad \frac{dT}{dt} = -\frac{n^2 T}{t}$$

$$\Rightarrow \quad \frac{dT}{T} = -\frac{n^2}{t}\,dt$$

$$\Rightarrow \quad \ln|T| = -n^2 \ln|t| \Rightarrow T(t) = T_n(t) = C_n t^{-n^2}$$

$$\Rightarrow \quad u_n(x,t) = X_n(x)\,T_n(t) = \underbrace{B_n C_n}_{=b_n} t^{-n^2} \sin(nx)$$

Observera att $u_0 = 0$ och att u_n och u_{-n} är linjärt beroende.

v.g.v.

$$u(x,t) = \sum_{n=1}^{\infty} b_n t^{-n^2} \sin(nx)$$

löser (I) + (RV)

$$u(x,1) = \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$= \{(BV)\} = 4\sin^3(x)$$

$$= \{\text{BETA s. } 128\} = 3\sin(x) - \sin(3x)$$

$$\Rightarrow \quad b_n = 0, \quad n \neq 1, 3. \quad b_1 = 3, \quad b_3 = -1$$

Svar: $u(x,t) = 3t^{-1}\sin(x) - t^{-9}\sin(3x)$

2007-12-14.2

$\{a_n\}_{n=0}^{\infty}$ är en talföljd som

uppfyller $a_0=4$, $a_1=10$ och

$$a_{n+2}=2a_n+a_{n+1}+10\cdot 4^n$$

för $n=0,1,2,\ldots$. Bestäm talföljden.

Lö Vi använder z-transformen.

BETA s. 327 och 328,

Z4 $\quad a_{n+k} \xrightarrow{Z} z^k A(z)-z^k a_0-z^{k-1}a_1-\ldots$

$-z a_{k-1}$

Z10 $\quad a^n \xrightarrow{Z} \dfrac{z}{z-a}$

ger r.g.v.

$$z^2 A(z) - 4z^2 - 10z = 2A(z) + zA(z) - 4z + 10\frac{z}{z-4}$$

$$\Rightarrow \quad A(z)\underbrace{(z^2 - z - 2)}_{=(z+1)(z-2)} = 4z^2 + 6z + 10\frac{z}{z-4}$$

$$\Rightarrow \quad A(z) = z \cdot \frac{(4z+6)(z-4)+10}{(z+1)(z-2)(z-4)}$$

$$= z \cdot \frac{4z^2 - 10z - 14}{(z+1)(z-2)(z-4)} =$$

$z = -1$ är en rot till täljaren

$$\begin{array}{l} \quad 4z - 14 \\ \hline 4z^2 - 10z - 14 \;\big|\; z+1 \\ -(4z^2 + 4z) \\ \hline \quad -14z - 14 \\ -(-14z - 14) \\ \hline \quad\; 0 \quad\;\; 0 \end{array}$$

v. g. v.

$$= z \cdot \frac{4z-14}{(z-4)(z-2)} =$$

Partialbråksuppdelning

$$= \frac{4z-14}{(z-4)(z-2)} = \frac{1}{z-4} + \frac{3}{z-2} =$$

$$= \frac{z}{z-4} + 3\frac{z}{z-2}$$

BETA s. 328.

z 10 $\frac{z}{z-a} \xrightarrow{Z^{-1}} a^n$

ger

$$a_n = 4^n + 3 \cdot 2^n$$

V.S.V.

Svar: $\{a_n\}_{n=0}^{\infty} = \{4^n + 3\cdot 2^n\}_{n=0}^{\infty}$

Verifiering: $a_0 = 1+3 = 4$ ok!

$a_1 = 4^1 + 3\cdot 2^1 = 10$ ok!

$a_{n+2} = 4^{n+2} + 3\cdot 2^{n+2}$

$a_{n+1} + 2a_n + 10\cdot 4^n = 4^{n+1} + 3\cdot 2^{n+1} + 2(4^n + 3\cdot 2^n) + 10\cdot 4^n =$

$= 4^{n+1} + 3\cdot 2^{n+1} + 2\cdot 4^n + 3\cdot 2^{n+1} + 2\cdot 4^n + 2\cdot 4^{n+1}$

$= 4^{n+1} + 6\cdot 2^{n+1} + 4\cdot 4^n + 2\cdot 4^{n+1} =$

$= 4^{n+1} + 3\cdot 2^{n+2} + 4^{n+1} + 2\cdot 4^{n+1} =$

$= 4\cdot 4^{n+1} + 3\cdot 2^{n+2} = 4^{n+2} + 3\cdot 2^{n+2}$ ok!

2004-08-23.5

Beräkna första och andra distributionsderivatan till

$$f(t) = e^{-|t|}$$

Lr f kan skrivas som

$$f(t) = \theta(t)e^{-t} + (1-\theta(t))e^{t}$$

$$\Rightarrow f'(t) = \delta(t)e^{-t} - \theta(t)e^{-t} - \delta(t)e^{t} + (1-\theta(t))e^{t}$$

$$= -\theta(t)e^{-t} + (1-\theta(t))e^{t}$$

v.g.v.

$$f''(t) = -\delta(t)e^{-t} + \theta(t)e^{-t}$$

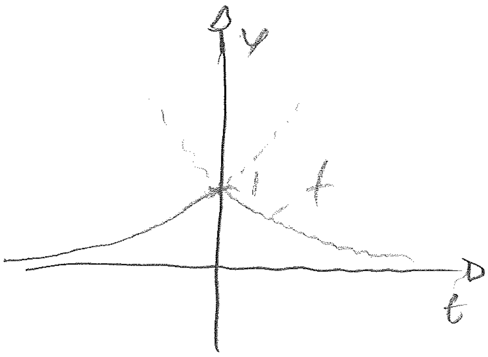
$$- \delta(t)e^{t} + (1-\theta(t))e^{t} =$$

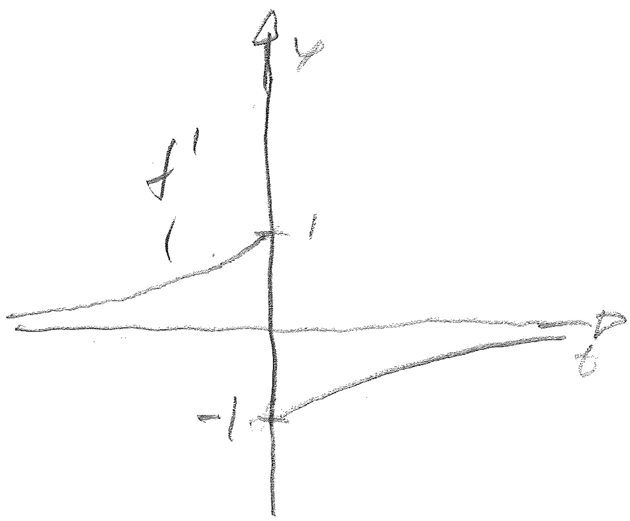
$$= \{ f(t)\delta(t) = f(0)\delta(t) \} =$$

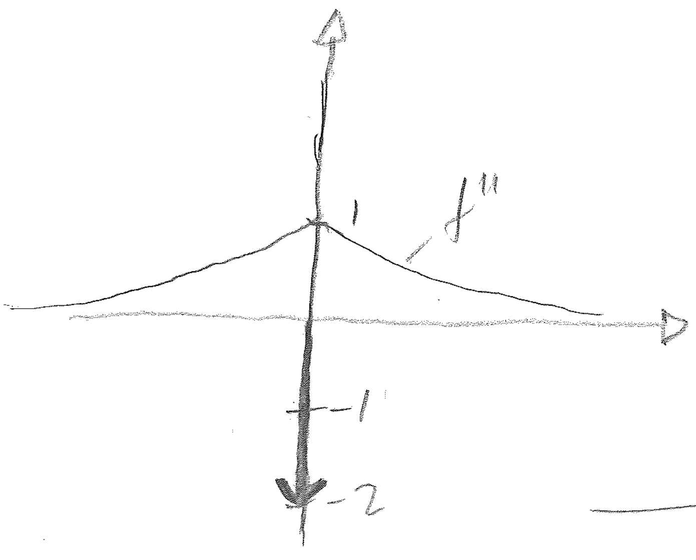
$$= e^{-|t|} - 2\delta(t)$$

Svar: $f'(t) = -\theta(t)e^{-t} + (1-\theta(t))e^{t}$

$f''(t) = e^{-|t|} - 2\delta(t)$







Slut övning 7